CHAPTER 1

INTRODUCTION TO MACHINERY PRINCIPLES

1.1 ELECTRICAL MACHINES, TRANSFORMERS, AND DAILY LIFE

An **electrical machine** is a device that can convert either mechanical energy to electrical energy or electrical energy to mechanical energy. When such a device is used to convert mechanical energy to electrical energy, it is called a *generator.* When it converts electrical energy to mechanical energy, it is called a *motor.* Since any given electrical machine can convert power in either direction, any machine can be used as either a generator or a motor. Almost all practical motors and generators convert energy from one form to another through the action of a magnetic field, and only machines using magnetic fields to perform such conversions are considered in this book.

The *transformer* is an electrical device that is closely related to electrical machines. It converts ac electrical energy at one voltage level to ac electrical energy at another voltage level. Since transformers operate on the same principles as generators and motors, depending on the action of a magnetic field to accomplish the change in voltage level, they are usually studied together with generators and motors.

These three types of electric devices are ubiquitous in modern daily life. Electric motors in the home run refrigerators, freezers, vacuum cleaners, blenders, air conditioners, fans, and many similar appliances. In the workplace, motors provide the motive power for almost all tools. Of course, generators are necessary to supply the power used by all these motors.

Why are electric motors and generators so common? The answer is very simple: Electric power is a clean and efficient energy source that is easy to transmit over long distances, and easy to control. An electric motor does not require constant ventilation and fuel the way that an internal-combustion engine does, so the motor is very well suited for use in environments where the pollutants associated with combustion are not desirable. Instead, heat or mechanical energy can be converted to electrical form at a distant location, the energy can be transmitted over long distances to the place where it is to be used, and it can be used cleanly in any home, office, or factory. Transformers aid this process by reducing the energy loss between the point of electric power generation and the point of its use.

1.2 A NOTE ON UNITS AND NOTATION

The design and study of electric machines and power systems are among the oldest areas of electrical engineering. Study began in the latter part of the nineteenth century. At that time, electrical units were being standardized internationally, and these units came to be universally used by engineers. Volts, amperes, ohms, watts, and similar units, which are part of the metric system of units, have long been used to describe electrical quantities in machines.

In English-speaking countries, though, mechanical quantities had long been measured with the English system of units (inches, feet, pounds, etc.). This practice was followed in the study of machines. Therefore, for many years the electrical and mechanical quantities of machines have been measured with different systems of units.

In 1954, a comprehensive system of units based on the metric system was adopted as an international standard. This system of units became known as the *Système International* (SI) and has been adopted throughout most of the world. The United States is practically the sole holdout—even Britain and Canada have switched over to SI.

The SI units will inevitably become standard in the United States as time goes by, and professional societies such as the Institute of Electrical and Electronics Engineers (IEEE) have standardized on metric units for all work. However, many people have grown up using English units, and this system will remain in daily use for a long time. Engineering students and working engineers in the United States today must be familiar with both sets of units, since they will encounter both throughout their professional lives. Therefore, this book includes problems and examples using both SI and English units. The emphasis in the examples is on SI units, but the older system is not entirely neglected.

Notation

In this book, vectors, electrical phasors, and other complex values are shown in bold face (e.g., **F**), while scalars are shown in italic face (e.g., *R*). In addition, a special font is used to represent magnetic quantities such as magnetomotive force (e.g., $\mathcal{F}$).

1.3 ROTATIONAL MOTION, NEWTON'S LAW, AND POWER RELATIONSHIPS

Almost all electric machines rotate about an axis, called the *shaft* of the machine. Because of the rotational nature of machinery, it is important to have a basic understanding of rotational motion. This section contains a brief review of the concepts of distance, velocity, acceleration, Newton's law, and power as they apply to rotating machinery. For a more detailed discussion of the concepts of rotational dynamics, see References 2, 4, and 5.

In general, a three-dimensional vector is required to completely describe the rotation of an object in space. However, machines normally turn on a fixed shaft, so their rotation is restricted to one angular dimension. Relative to a given end of the machine's shaft, the direction of rotation can be described as either *clockwise* (CW) or *counterclockwise* (CCW). For the purpose of this volume, a counterclockwise angle of rotation is assumed to be positive, and a clockwise one is assumed to be negative. For rotation about a fixed shaft, all the concepts in this section reduce to scalars.

Each major concept of rotational motion is defined below and is related to the corresponding idea from linear motion.

Angular Position θ

The angular position θ of an object is the angle at which it is oriented, measured from some arbitrary reference point. Angular position is usually measured in radians or degrees. It corresponds to the linear concept of distance along a line.

Angular Velocity ω

Angular velocity (or speed) is the rate of change in angular position with respect to time. It is assumed positive if the rotation is in a counterclockwise direction. Angular velocity is the rotational analog of the concept of velocity on a line. One-dimensional linear velocity along a line is defined as the rate of change of the displacement along the line (r) with respect to time

$$v = \frac{dr}{dt} \tag{1-1}$$

Similarly, angular velocity ω is defined as the rate of change of the angular displacement θ with respect to time.

$$\omega = \frac{d\theta}{dt} \tag{1-2}$$

If the units of angular position are radians, then angular velocity is measured in radians per second.

In dealing with ordinary electric machines, engineers often use units other than radians per second to describe shaft speed. Frequently, the speed is given in revolutions per second or revolutions per minute. Because speed is such an important quantity in the study of machines, it is customary to use different symbols for speed when it is expressed in different units. By using these different symbols, any possible confusion as to the units intended is minimized. The following symbols are used in this book to describe angular velocity:

ω_m	angular velocity expressed in radians per second
f_m	angular velocity expressed in revolutions per second
n_m	angular velocity expressed in revolutions per minute

The subscript m on these symbols indicates a mechanical quantity, as opposed to an electrical quantity. If there is no possibility of confusion between mechanical and electrical quantities, the subscript is often left out.

These measures of shaft speed are related to each other by the following equations:

$$n_m = 60f_m \tag{1–3a}$$

$$f_m = \frac{\omega_m}{2\pi} \tag{1–3b}$$

Angular Acceleration α

Angular acceleration is the rate of change in angular velocity with respect to time. It is assumed positive if the angular velocity is increasing in an algebraic sense. Angular acceleration is the rotational analog of the concept of acceleration on a line. Just as one-dimensional linear acceleration is defined by the equation

$$a = \frac{dv}{dt} \tag{1–4}$$

angular acceleration is defined by

$$\alpha = \frac{d\omega}{dt} \tag{1–5}$$

If the units of angular velocity are radians per second, then angular acceleration is measured in radians per second squared.

Torque τ

In linear motion, a *force* applied to an object causes its velocity to change. In the absence of a net force on the object, its velocity is constant. The greater the force applied to the object, the more rapidly its velocity changes.

There exists a similar concept for rotation. When an object is rotating, its angular velocity is constant unless a *torque* is present on it. The greater the torque on the object, the more rapidly the angular velocity of the object changes.

What is torque? It can loosely be called the "twisting force" on an object. Intuitively, torque is fairly easy to understand. Imagine a cylinder that is free to

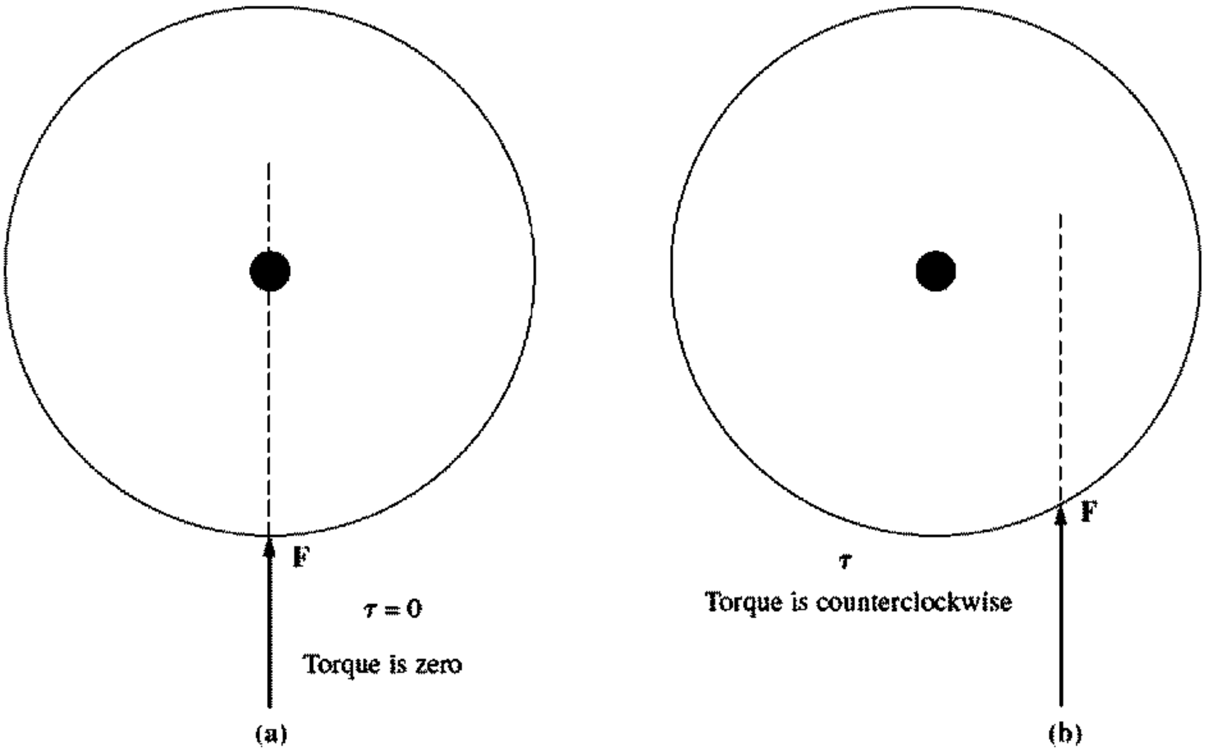


FIGURE 1–1
(a) A force applied to a cylinder so that it passes through the axis of rotation. $\tau = 0$. (b) A force applied to a cylinder so that its line of action misses the axis of rotation. Here τ is counterclockwise.

rotate about its axis. If a force is applied to the cylinder in such a way that its line of action passes through the axis (Figure 1–1a), then the cylinder will not rotate. However, if the same force is placed so that its line of action passes to the right of the axis (Figure 1–1b), then the cylinder will tend to rotate in a counterclockwise direction. The torque or twisting action on the cylinder depends on (1) the magnitude of the applied force and (2) the distance between the axis of rotation and the line of action of the force.

The torque on an object is defined as the product of the force applied to the object and the smallest distance between the line of action of the force and the object's axis of rotation. If **r** is a vector pointing from the axis of rotation to the point of application of the force, and if **F** is the applied force, then the torque can be described as

$$\begin{aligned} \tau &= (\text{force applied})(\text{perpendicular distance}) \\ &= (F)\,(r \sin \theta) \\ &= rF \sin \theta \end{aligned} \tag{1–6}$$

where θ is the angle between the vector **r** and the vector **F**. The direction of the torque is clockwise if it would tend to cause a clockwise rotation and counterclockwise if it would tend to cause a counterclockwise rotation (Figure 1–2).

The units of torque are newton-meters in SI units and pound-feet in the English system.

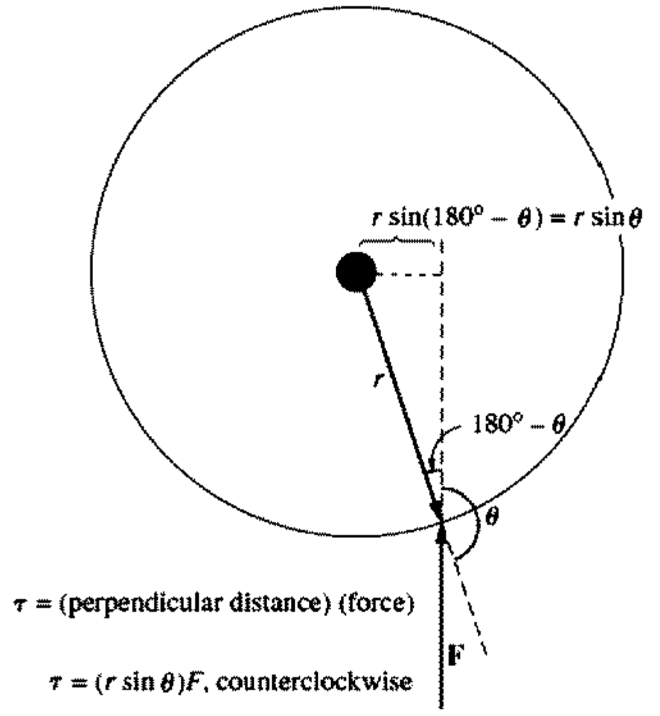


FIGURE 1–2
Derivation of the equation for the torque on an object.

Newton's Law of Rotation

Newton's law for objects moving along a straight line describes the relationship between the force applied to an object and its resulting acceleration. This relationship is given by the equation

$$F = ma \tag{1–7}$$

where

F = net force applied to an object
m = mass of the object
a = resulting acceleration

In SI units, force is measured in newtons, mass in kilograms, and acceleration in meters per second squared. In the English system, force is measured in pounds, mass in slugs, and acceleration in feet per second squared.

A similar equation describes the relationship between the torque applied to an object and its resulting angular acceleration. This relationship, called *Newton's law of rotation,* is given by the equation

$$\tau = J\alpha \tag{1–8}$$

where τ is the net applied torque in newton-meters or pound-feet and α is the resulting angular acceleration in radians per second squared. The term J serves the same purpose as an object's mass in linear motion. It is called the *moment of inertia* of the object and is measured in kilogram-meters squared or slug-feet squared. Calculation of the moment of inertia of an object is beyond the scope of this book. For information about it see Ref. 2.

Work W

For linear motion, work is defined as the application of a *force* through a *distance*. In equation form,

$$W = \int F\,dr \tag{1–9}$$

where it is assumed that the force is collinear with the direction of motion. For the special case of a constant force applied collinearly with the direction of motion, this equation becomes just

$$W = Fr \tag{1–10}$$

The units of work are joules in SI and foot-pounds in the English system.

For rotational motion, work is the application of a *torque* through an *angle*. Here the equation for work is

$$W = \int \tau\,d\theta \tag{1–11}$$

and if the torque is constant,

$$W = \tau\theta \tag{1–12}$$

Power P

Power is the rate of doing work, or the increase in work per unit time. The equation for power is

$$P = \frac{dW}{dt} \tag{1–13}$$

It is usually measured in joules per second (watts), but also can be measured in foot-pounds per second or in horsepower.

By this definition, and assuming that force is constant and collinear with the direction of motion, power is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(Fr) = F\left(\frac{dr}{dt}\right) = Fv \tag{1–14}$$

Similarly, assuming constant torque, power in rotational motion is given by

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\left(\frac{d\theta}{dt}\right) = \tau\omega$$

$$P = \tau\omega \tag{1–15}$$

Equation (1–15) is very important in the study of electric machinery, because it can describe the mechanical power on the shaft of a motor or generator.

Equation (1–15) is the correct relationship among power, torque, and speed if power is measured in watts, torque in newton-meters, and speed in radians per second. If other units are used to measure any of the above quantities, then a constant must be introduced into the equation for unit conversion factors. It is still common in U.S. engineering practice to measure torque in pound-feet, speed in revolutions per minute, and power in either watts or horsepower. If the appropriate conversion factors are included in each term, then Equation (1–15) becomes

$$P\,(\text{watts}) = \frac{\tau\,(\text{lb-ft})\; n\,(\text{r/min})}{7.04} \tag{1–16}$$

$$P\,(\text{horsepower}) = \frac{\tau\,(\text{lb-ft})\; n\,(\text{r/min})}{5252} \tag{1–17}$$

where torque is measured in pound-feet and speed is measured in revolutions per minute.

1.4 THE MAGNETIC FIELD

As previously stated, magnetic fields are the fundamental mechanism by which energy is converted from one form to another in motors, generators, and transformers. Four basic principles describe how magnetic fields are used in these devices:

1. A current-carrying wire produces a magnetic field in the area around it.
2. A time-changing magnetic field induces a voltage in a coil of wire if it passes through that coil. (This is the basis of *transformer action*.)
3. A current-carrying wire in the presence of a magnetic field has a force induced on it. (This is the basis of *motor action*.)
4. A moving wire in the presence of a magnetic field has a voltage induced in it. (This is the basis of *generator action*.)

This section describes and elaborates on the production of a magnetic field by a current-carrying wire, while later sections of this chapter explain the remaining three principles.

Production of a Magnetic Field

The basic law governing the production of a magnetic field by a current is Ampere's law:

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{net}} \tag{1–18}$$

where **H** is the magnetic field intensity produced by the current I_{net}, and dl is a differential element of length along the path of integration. In SI units, I is measured in amperes and H is measured in ampere-turns per meter. To better understand the meaning of this equation, it is helpful to apply it to the simple example in Figure 1–3. Figure 1–3 shows a rectangular core with a winding of N turns of wire wrapped about one leg of the core. If the core is composed of iron or certain other similar metals (collectively called *ferromagnetic materials*), essentially all the magnetic field produced by the current will remain inside the core, so the path of integration in Ampere's law is the mean path length of the core l_c. The current

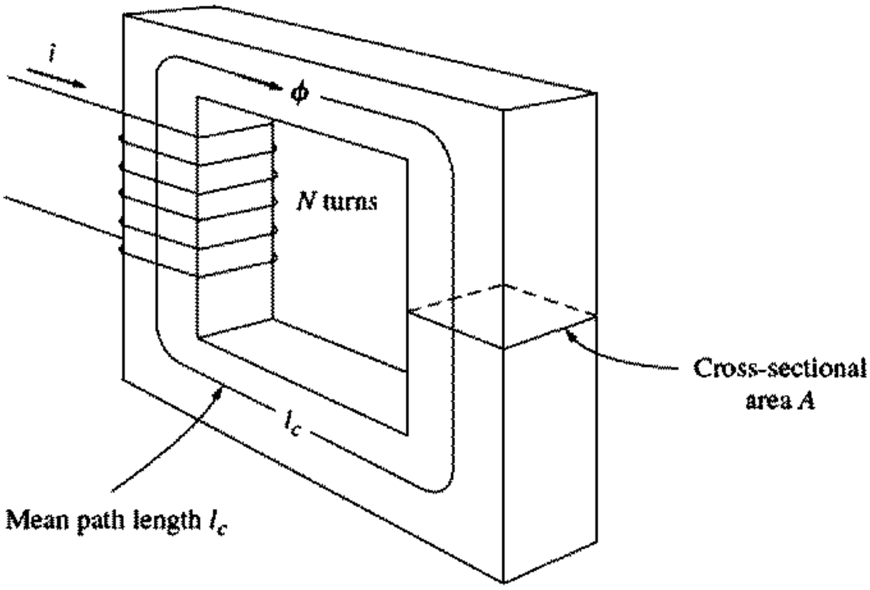


FIGURE 1–3
A simple magnetic core.

passing within the path of integration I_{net} is then Ni, since the coil of wire cuts the path of integration N times while carrying current i. Ampere's law thus becomes

$$Hl_c = Ni \tag{1–19}$$

Here H is the magnitude of the magnetic field intensity vector **H**. Therefore, the magnitude of the magnetic field intensity in the core due to the applied current is

$$H = \frac{Ni}{l_c} \tag{1–20}$$

The magnetic field intensity **H** is in a sense a measure of the "effort" that a current is putting into the establishment of a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. The relationship between the magnetic field intensity **H** and the resulting magnetic flux density **B** produced within a material is given by

$$\mathbf{B} = \mu\mathbf{H} \tag{1–21}$$

where

$\mathbf{H}$ = magnetic field intensity
μ = magnetic *permeability* of material
$\mathbf{B}$ = resulting magnetic flux density produced

The actual magnetic flux density produced in a piece of material is thus given by a product of two terms:

H, representing the effort exerted by the current to establish a magnetic field

μ, representing the relative ease of establishing a magnetic field in a given material

The units of magnetic field intensity are ampere-turns per meter, the units of permeability are henrys per meter, and the units of the resulting flux density are webers per square meter, known as teslas (T).

The permeability of free space is called μ_0, and its value is

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \tag{1–22}$$

The permeability of any other material compared to the permeability of free space is called its *relative permeability:*

$$\mu_r = \frac{\mu}{\mu_0} \tag{1–23}$$

Relative permeability is a convenient way to compare the magnetizability of materials. For example, the steels used in modern machines have relative permeabilities of 2000 to 6000 or even more. This means that, for a given amount of current, 2000 to 6000 times more flux is established in a piece of steel than in a corresponding area of air. (The permeability of air is essentially the same as the permeability of free space.) Obviously, the metals in a transformer or motor core play an extremely important part in increasing and concentrating the magnetic flux in the device.

Also, because the permeability of iron is so much higher than that of air, the great majority of the flux in an iron core like that in Figure 1–3 remains inside the core instead of traveling through the surrounding air, which has much lower permeability. The small leakage flux that does leave the iron core is very important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.

In a core such as the one shown in Figure 1–3, the magnitude of the flux density is given by

$$B = \mu H = \frac{\mu N i}{l_c} \tag{1–24}$$

Now the total flux in a given area is given by

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{A} \tag{1–25a}$$

where $d\mathbf{A}$ is the differential unit of area. If the flux density vector is perpendicular to a plane of area A, and if the flux density is constant throughout the area, then this equation reduces to

$$\phi = BA \tag{1–25b}$$

Thus, the total flux in the core in Figure 1–3 due to the current i in the winding is

$$\phi = BA = \frac{\mu N i A}{l_c} \tag{1–26}$$

where A is the cross-sectional area of the core.

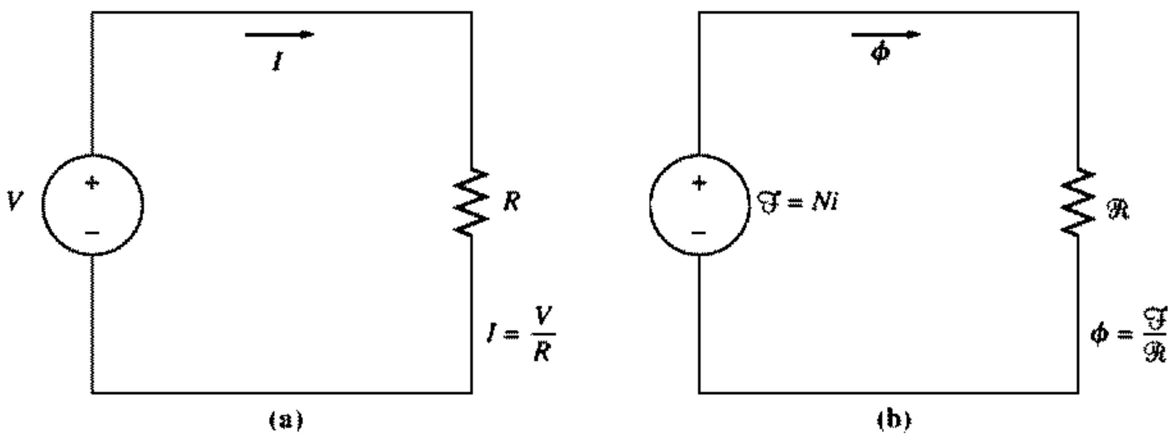


FIGURE 1–4
(a) A simple electric circuit. (b) The magnetic circuit analog to a transformer core.

Magnetic Circuits

In Equation (1–26) we see that the *current* in a coil of wire wrapped around a core produces a magnetic flux in the core. This is in some sense analogous to a voltage in an electric circuit producing a current flow. It is possible to define a "magnetic circuit" whose behavior is governed by equations analogous to those for an electric circuit. The magnetic circuit model of magnetic behavior is often used in the design of electric machines and transformers to simplify the otherwise quite complex design process.

In a simple electric circuit such as the one shown in Figure 1–4a, the voltage source V drives a current I around the circuit through a resistance R. The relationship between these quantities is given by Ohm's law:

$$V = IR$$

In the electric circuit, it is the voltage or electromotive force that drives the current flow. By analogy, the corresponding quantity in the magnetic circuit is called the *magnetomotive force* (mmf). The magnetomotive force of the magnetic circuit is equal to the effective current flow applied to the core, or

$$\mathcal{F} = Ni \tag{1–27}$$

where $\mathcal{F}$ is the symbol for magnetomotive force, measured in ampere-turns.

Like the voltage source in the electric circuit, the magnetomotive force in the magnetic circuit has a polarity associated with it. The *positive* end of the mmf source is the end from which the flux exits, and the *negative* end of the mmf source is the end at which the flux reenters. The polarity of the mmf from a coil of wire can be determined from a modification of the right-hand rule: If the fingers of the right hand curl in the direction of the current flow in a coil of wire, then the thumb will point in the direction of the positive mmf (see Figure 1–5).

In an electric circuit, the applied voltage causes a current I to flow. Similarly, in a magnetic circuit, the applied magnetomotive force causes flux ϕ to be produced. The relationship between voltage and current in an electric circuit is

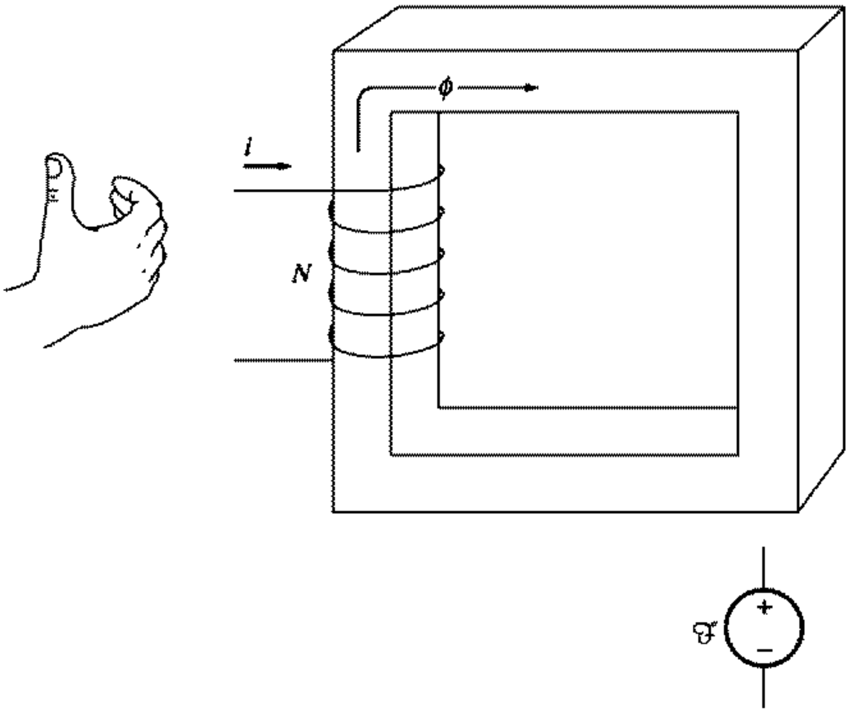


FIGURE 1–5
Determining the polarity of a magnetomotive force source in a magnetic circuit.

Ohm's law ($V = IR$); similarly, the relationship between magnetomotive force and flux is

$$\boxed{\mathcal{F} = \phi \mathcal{R}} \tag{1–28}$$

where

$\mathcal{F}$ = magnetomotive force of circuit

ϕ = flux of circuit

$\mathcal{R}$ = *reluctance* of circuit

The reluctance of a magnetic circuit is the counterpart of electrical resistance, and its units are ampere-turns per weber.

There is also a magnetic analog of conductance. Just as the conductance of an electric circuit is the reciprocal of its resistance, the *permeance* $\mathcal{P}$ of a magnetic circuit is the reciprocal of its reluctance:

$$\mathcal{P} = \frac{1}{\mathcal{R}} \tag{1–29}$$

The relationship between magnetomotive force and flux can thus be expressed as

$$\phi = \mathcal{F}\mathcal{P} \tag{1–30}$$

Under some circumstances, it is easier to work with the permeance of a magnetic circuit than with its reluctance.

What is the reluctance of the core in Figure 1–3? The resulting flux in this core is given by Equation (1–26):

$$\phi = BA = \frac{\mu NiA}{l_c} \tag{1–26}$$

$$= Ni\left(\frac{\mu A}{l_c}\right)$$

$$\phi = \mathcal{F}\left(\frac{\mu A}{l_c}\right) \tag{1–31}$$

By comparing Equation (1–31) with Equation (1–28), we see that the reluctance of the core is

$$\mathcal{R} = \frac{l_c}{\mu A} \tag{1–32}$$

Reluctances in a magnetic circuit obey the same rules as resistances in an electric circuit. The equivalent reluctance of a number of reluctances in series is just the sum of the individual reluctances:

$$\mathcal{R}_{eq} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \cdots \tag{1–33}$$

Similarly, reluctances in parallel combine according to the equation

$$\frac{1}{\mathcal{R}_{eq}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \cdots \tag{1–34}$$

Permeances in series and parallel obey the same rules as electrical conductances.

Calculations of the flux in a core performed by using the magnetic circuit concepts are *always* approximations—at best, they are accurate to within about 5 percent of the real answer. There are a number of reasons for this inherent inaccuracy:

1. The magnetic circuit concept assumes that all flux is confined within a magnetic core. Unfortunately, this is not quite true. The permeability of a ferromagnetic core is 2000 to 6000 times that of air, but a small fraction of the flux escapes from the core into the surrounding low-permeability air. This flux outside the core is called *leakage flux*, and it plays a very important role in electric machine design.
2. The calculation of reluctance assumes a certain mean path length and cross-sectional area for the core. These assumptions are not really very good, especially at corners.
3. In ferromagnetic materials, the permeability varies with the amount of flux already in the material. This nonlinear effect is described in detail. It adds yet another source of error to magnetic circuit analysis, since the reluctances used in magnetic circuit calculations depend on the permeability of the material.

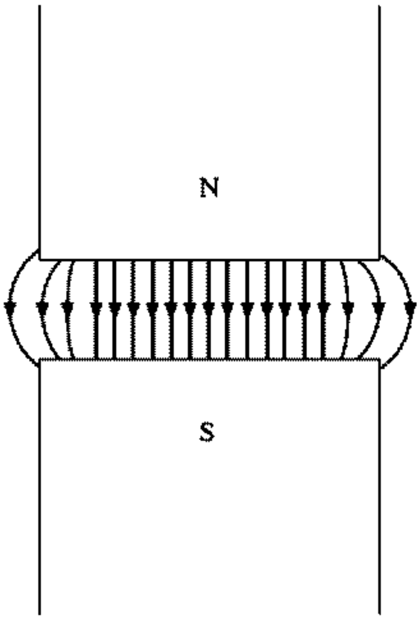


FIGURE 1–6
The fringing effect of a magnetic field at an air gap. Note the increased cross-sectional area of the air gap compared with the cross-sectional area of the metal.

4. If there are air gaps in the flux path in a core, the effective cross-sectional area of the air gap will be larger than the cross-sectional area of the iron core on either side. The extra effective area is caused by the "fringing effect" of the magnetic field at the air gap (Figure 1–6).

It is possible to partially offset these inherent sources of error by using a "corrected" or "effective" mean path length and the cross-sectional area instead of the actual physical length and area in the calculations.

There are many inherent limitations to the concept of a magnetic circuit, but it is still the easiest design tool available for calculating fluxes in practical machinery design. Exact calculations using Maxwell's equations are just too difficult, and they are not needed anyway, since satisfactory results may be achieved with this approximate method.

The following examples illustrate basic magnetic circuit calculations. Note that in these examples the answers are given to three significant digits.

Example 1–1. A ferromagnetic core is shown in Figure 1–7a. Three sides of this core are of uniform width, while the fourth side is somewhat thinner. The depth of the core (into the page) is 10 cm, and the other dimensions are shown in the figure. There is a 200-turn coil wrapped around the left side of the core. Assuming relative permeability μ_r of 2500, how much flux will be produced by a 1-A input current?

Solution

We will solve this problem twice, once by hand and once by a MATLAB program, and show that both approaches yield the same answer.

Three sides of the core have the same cross-sectional areas, while the fourth side has a different area. Thus, the core can be divided into two regions: (1) the single thinner side and (2) the other three sides taken together. The magnetic circuit corresponding to this core is shown in Figure 1–7b.

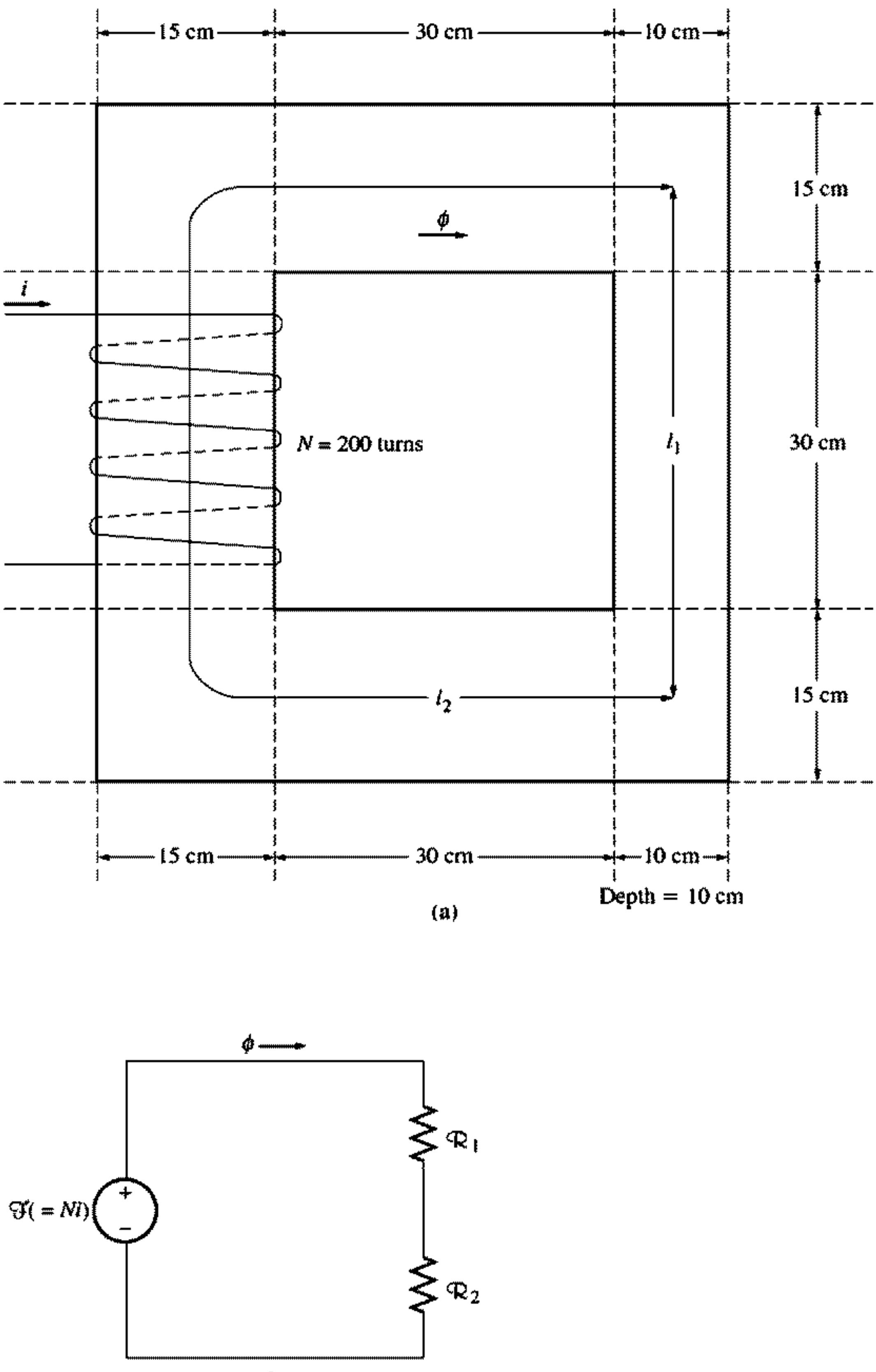


FIGURE 1–7
(a) The ferromagnetic core of Example 1–1. (b) The magnetic circuit corresponding to (a).

The mean path length of region 1 is 45 cm, and the cross-sectional area is 10 × 10 cm = 100 cm^2. Therefore, the reluctance in the first region is

$$\mathcal{R}_1 = \frac{l_1}{\mu A_1} = \frac{l_1}{\mu_r \mu_0 A_1} \tag{1–32}$$

$$= \frac{0.45 \text{ m}}{(2500)(4\pi \times 10^{-7})(0.01 \text{ m}^2)}$$

$$= 14{,}300 \text{ A} \cdot \text{turns/Wb}$$

The mean path length of region 2 is 130 cm, and the cross-sectional area is 15 × 10 cm = 150 cm^2. Therefore, the reluctance in the second region is

$$\mathcal{R}_2 = \frac{l_2}{\mu A_2} = \frac{l_2}{\mu_r \mu_0 A_2} \tag{1–32}$$

$$= \frac{1.3 \text{ m}}{(2500)(4\pi \times 10^{-7})(0.015 \text{ m}^2)}$$

$$= 27{,}600 \text{ A} \cdot \text{turns/Wb}$$

Therefore, the total reluctance in the core is

$$\mathcal{R}_{\text{eq}} = \mathcal{R}_1 + \mathcal{R}_2$$

$$= 14{,}300 \text{ A} \cdot \text{turns/Wb} + 27{,}600 \text{ A} \cdot \text{turns/Wb}$$

$$= 41{,}900 \text{ A} \cdot \text{turns/Wb}$$

The total magnetomotive force is

$$\mathcal{F} = Ni = (200 \text{ turns})(1.0 \text{ A}) = 200 \text{ A} \cdot \text{turns}$$

The total flux in the core is given by

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{200 \text{ A} \cdot \text{turns}}{41{,}900 \text{ A} \cdot \text{turns/Wb}}$$

$$= 0.0048 \text{ Wb}$$

This calculation can be performed by using a MATLAB script file, if desired. A simple script to calculate the flux in the core is shown below.

```
% M-file: ex1_1.m
% M-file to calculate the flux in Example 1-1.
l1 = 0.45;                  % Length of region 1
l2 = 1.3;                   % Length of region 2
a1 = 0.01;                  % Area of region 1
a2 = 0.015;                 % Area of region 2
ur = 2500;                  % Relative permeability
u0 = 4*pi*1E-7;             % Permeability of free space
n = 200;                    % Number of turns on core
i = 1;                      % Current in amps

% Calculate the first reluctance
r1 = l1 / (ur * u0 * a1);
disp (['r1 = ' num2str(r1)]);

% Calculate the second reluctance
r2 = l2 / (ur * u0 * a2);
disp (['r2 = ' num2str(r2)]);
```

```
% Calculate the total reluctance
rtot = r1 + r2;

% Calculate the mmf
mmf = n * i;

% Finally, get the flux in the core
flux = mmf / rtot;

% Display result
disp (['Flux = ' num2str(flux)]);
```

When this program is executed, the results are:

```
» ex1_1
r1 = 14323.9449
r2 = 27586.8568
Flux = 0.004772
```

This program produces the same answer as our hand calculations to the number of significant digits in the problem.

Example 1–2. Figure 1–8a shows a ferromagnetic core whose mean path length is 40 cm. There is a small gap of 0.05 cm in the structure of the otherwise whole core. The cross-sectional area of the core is 12 cm^2, the relative permeability of the core is 4000, and the coil of wire on the core has 400 turns. Assume that fringing in the air gap increases the effective cross-sectional area of the air gap by 5 percent. Given this information, find (a) the total reluctance of the flux path (iron plus air gap) and (b) the current required to produce a flux density of 0.5 T in the air gap.

Solution

The magnetic circuit corresponding to this core is shown in Figure 1–8b.

(a) The reluctance of the core is

$$\mathcal{R}_c = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_r \mu_0 A_c} \tag{1–32}$$

$$= \frac{0.4 \text{ m}}{(4000)(4\pi \times 10^{-7})(0.002 \text{ m}^2)}$$

$$= 66{,}300 \text{ A} \cdot \text{turns/Wb}$$

The effective area of the air gap is $1.05 \times 12 \text{ cm}^2 = 12.6 \text{ cm}^2$, so the reluctance of the air gap is

$$\mathcal{R}_a = \frac{l_a}{\mu_0 A_a} \tag{1–32}$$

$$= \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7})(0.00126 \text{ m}^2)}$$

$$= 316{,}000 \text{ A} \cdot \text{turns/Wb}$$

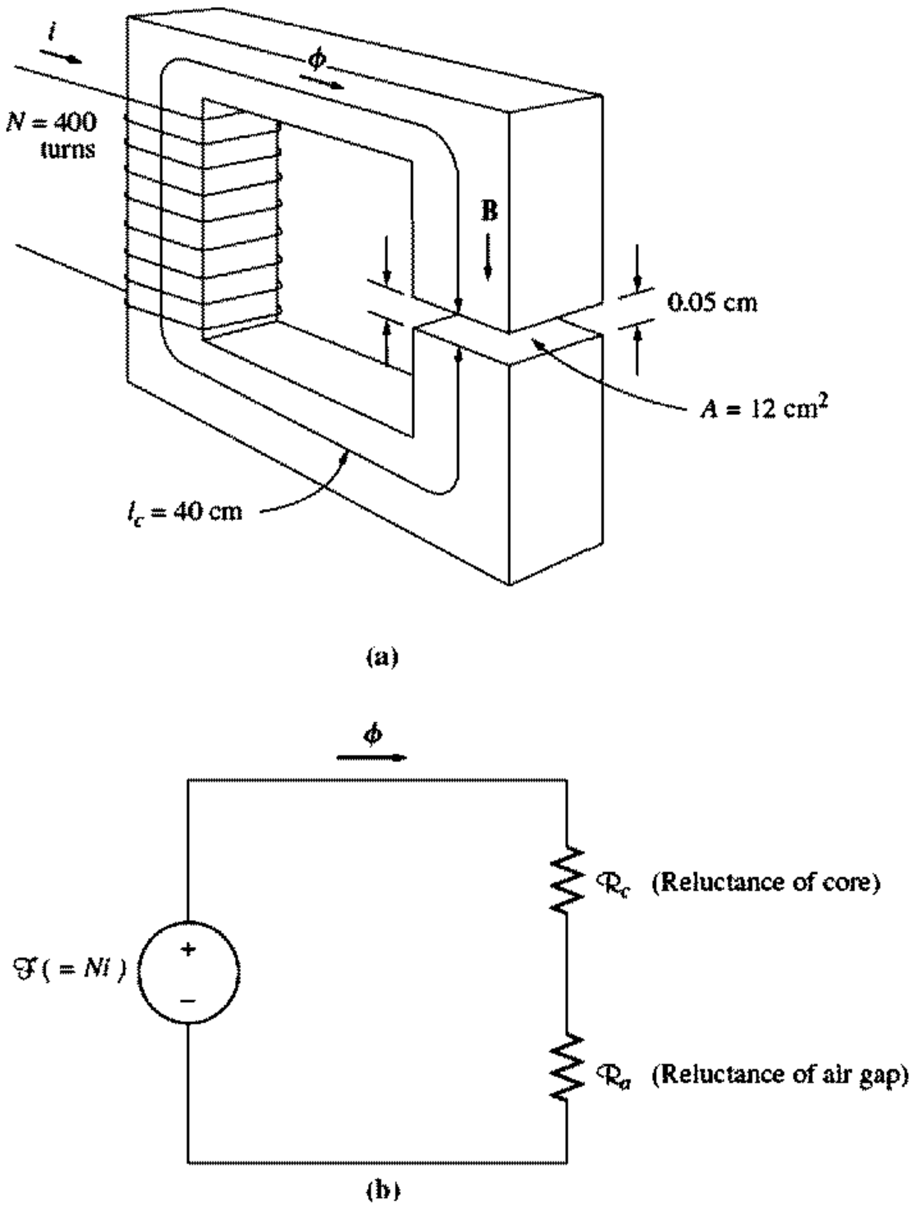


FIGURE 1–8
(a) The ferromagnetic core of Example 1–2. (b) The magnetic circuit corresponding to (a).

Therefore, the total reluctance of the flux path is

$$\begin{aligned}\mathcal{R}_{eq} &= \mathcal{R}_c + \mathcal{R}_a \\ &= 66{,}300 \text{ A} \cdot \text{turns/Wb} + 316{,}000 \text{ A} \cdot \text{turns/Wb} \\ &= 382{,}300 \text{ A} \cdot \text{turns/Wb}\end{aligned}$$

Note that the air gap contributes most of the reluctance even though it is 800 times shorter than the core.

(b) Equation (1–28) states that

$$\mathcal{F} = \phi\mathcal{R} \tag{1–28}$$

Since the flux $\phi = BA$ and $\mathcal{F} = Ni$, this equation becomes

$$Ni = BA\mathcal{R}$$

so

$$i = \frac{BA\mathcal{R}}{N}$$

$$= \frac{(0.5\ \mathrm{T})(0.00126\ \mathrm{m}^2)(383{,}200\ \mathrm{A \cdot turns/Wb})}{400\ \mathrm{turns}}$$

$$= 0.602\ \mathrm{A}$$

Notice that, since the *air-gap* flux was required, the effective air-gap area was used in the above equation.

Example 1–3. Figure 1–9a shows a simplified rotor and stator for a dc motor. The mean path length of the stator is 50 cm, and its cross-sectional area is 12 cm^2. The mean path length of the rotor is 5 cm, and its cross-sectional area also may be assumed to be 12 cm^2. Each air gap between the rotor and the stator is 0.05 cm wide, and the cross-sectional area of each air gap (including fringing) is 14 cm^2. The iron of the core has a relative permeability of 2000, and there are 200 turns of wire on the core. If the current in the wire is adjusted to be 1 A, what will the resulting flux density in the air gaps be?

Solution

To determine the flux density in the air gap, it is necessary to first calculate the magnetomotive force applied to the core and the total reluctance of the flux path. With this information, the total flux in the core can be found. Finally, knowing the cross-sectional area of the air gaps enables the flux density to be calculated.

The reluctance of the stator is

$$\mathcal{R}_s = \frac{l_s}{\mu_r \mu_0 A_s}$$

$$= \frac{0.5\ \mathrm{m}}{(2000)(4\pi \times 10^{-7})(0.0012\ \mathrm{m}^2)}$$

$$= 166{,}000\ \mathrm{A \cdot turns/Wb}$$

The reluctance of the rotor is

$$\mathcal{R}_r = \frac{l_r}{\mu_r \mu_0 A_r}$$

$$= \frac{0.05\ \mathrm{m}}{(2000)(4\pi \times 10^{-7})(0.0012\ \mathrm{m}^2)}$$

$$= 16{,}600\ \mathrm{A \cdot turns/Wb}$$

The reluctance of the air gaps is

$$\mathcal{R}_a = \frac{l_a}{\mu_r \mu_0 A_a}$$

$$= \frac{0.0005\ \mathrm{m}}{(1)(4\pi \times 10^{-7})(0.0014\ \mathrm{m}^2)}$$

$$= 284{,}000\ \mathrm{A \cdot turns/Wb}$$

The magnetic circuit corresponding to this machine is shown in Figure 1–9b. The total reluctance of the flux path is thus

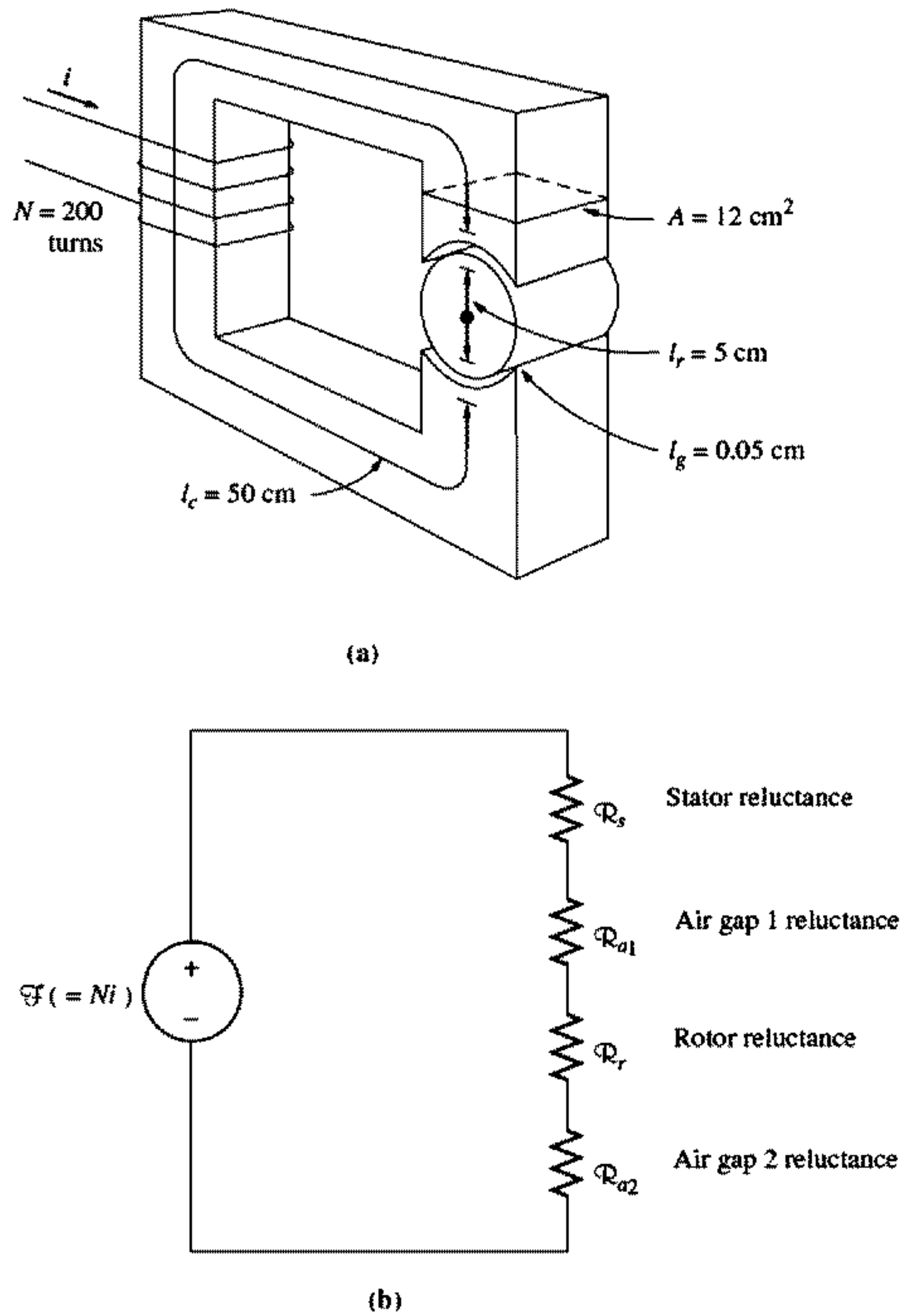


FIGURE 1–9
(a) A simplified diagram of a rotor and stator for a dc motor. (b) The magnetic circuit corresponding to (a).

$$\begin{aligned}\mathcal{R}_{eq} &= \mathcal{R}_s + \mathcal{R}_{a1} + \mathcal{R}_r + \mathcal{R}_{a2}\\ &= 166{,}000 + 284{,}000 + 16{,}600 + 284{,}000\ \text{A} \cdot \text{turns/Wb}\\ &= 751{,}000\ \text{A} \cdot \text{turns/Wb}\end{aligned}$$

The net magnetomotive force applied to the core is

$$\mathcal{F} = Ni = (200\ \text{turns})(1.0\ \text{A}) = 200\ \text{A} \cdot \text{turns}$$

Therefore, the total flux in the core is

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{200 \text{ A} \bullet \text{turns}}{751{,}000 \text{ A} \bullet \text{turns/Wb}}$$
$$= 0.00266 \text{ Wb}$$

Finally, the magnetic flux density in the motor's air gap is

$$B = \frac{\phi}{A} = \frac{0.000266 \text{ Wb}}{0.0014 \text{ m}^2} = 0.19 \text{ T}$$

Magnetic Behavior of Ferromagnetic Materials

Earlier in this section, magnetic permeability was defined by the equation

$$\mathbf{B} = \mu\mathbf{H} \tag{1–21}$$

It was explained that the permeability of ferromagnetic materials is very high, up to 6000 times the permeability of free space. In that discussion and in the examples that followed, the permeability was assumed to be constant regardless of the magnetomotive force applied to the material. Although permeability is constant in free space, this most certainly is *not* true for iron and other ferromagnetic materials.

To illustrate the behavior of magnetic permeability in a ferromagnetic material, apply a direct current to the core shown in Figure 1–3, starting with 0 A and slowly working up to the maximum permissible current. When the flux produced in the core is plotted versus the magnetomotive force producing it, the resulting plot looks like Figure 1–10a. This type of plot is called a *saturation curve* or a *magnetization curve.* At first, a small increase in the magnetomotive force produces a huge increase in the resulting flux. After a certain point, though, further increases in the magnetomotive force produce relatively smaller increases in the flux. Finally, an increase in the magnetomotive force produces almost no change at all. The region of this figure in which the curve flattens out is called the *saturation region,* and the core is said to be *saturated.* In contrast, the region where the flux changes very rapidly is called the *unsaturated region* of the curve, and the core is said to be *unsaturated.* The transition region between the unsaturated region and the saturated region is sometimes called the *knee* of the curve. Note that the flux produced in the core is linearly related to the applied magnetomotive force in the unsaturated region, and approaches a constant value regardless of magnetomotive force in the saturated region.

Another closely related plot is shown in Figure 1–10b. Figure 1–10b is a plot of magnetic flux density **B** versus magnetizing intensity **H**. From Equations (1–20) and (1–25b),

$$H = \frac{Ni}{l_c} = \frac{\mathcal{F}}{l_c} \tag{1–20}$$

$$\phi = BA \tag{1–25b}$$

it is easy to see that *magnetizing intensity is directly proportional to magnetomotive force* and *magnetic flux density is directly proportional to flux* for any given core. Therefore, the relationship between B and H has the same shape as the relationship

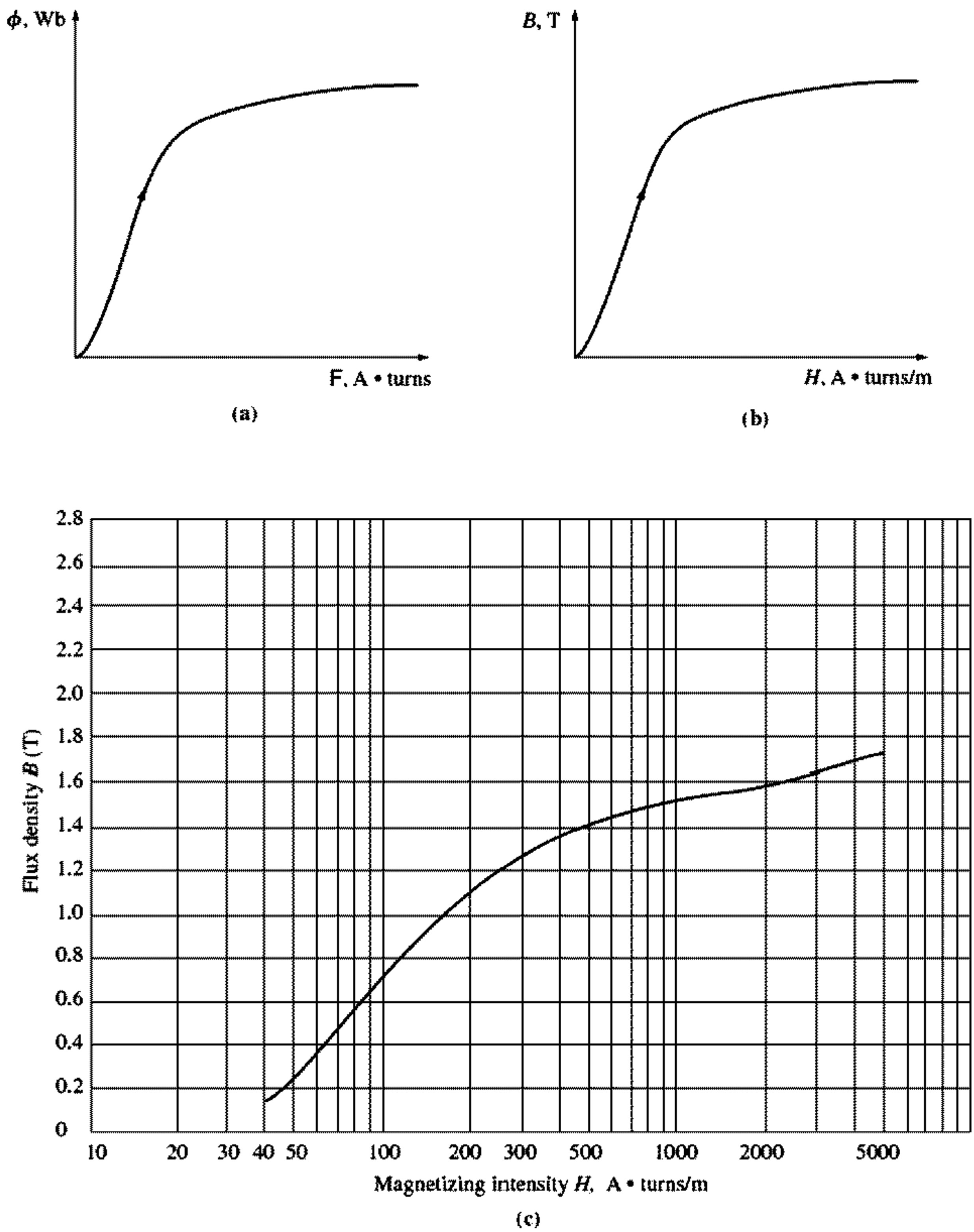


FIGURE 1–10
(a) Sketch of a dc magnetization curve for a ferromagnetic core. (b) The magnetization curve expressed in terms of flux density and magnetizing intensity. (c) A detailed magnetization curve for a typical piece of steel. (d) A plot of relative permeability μ_r as a function of magnetizing intensity H for a typical piece of steel.

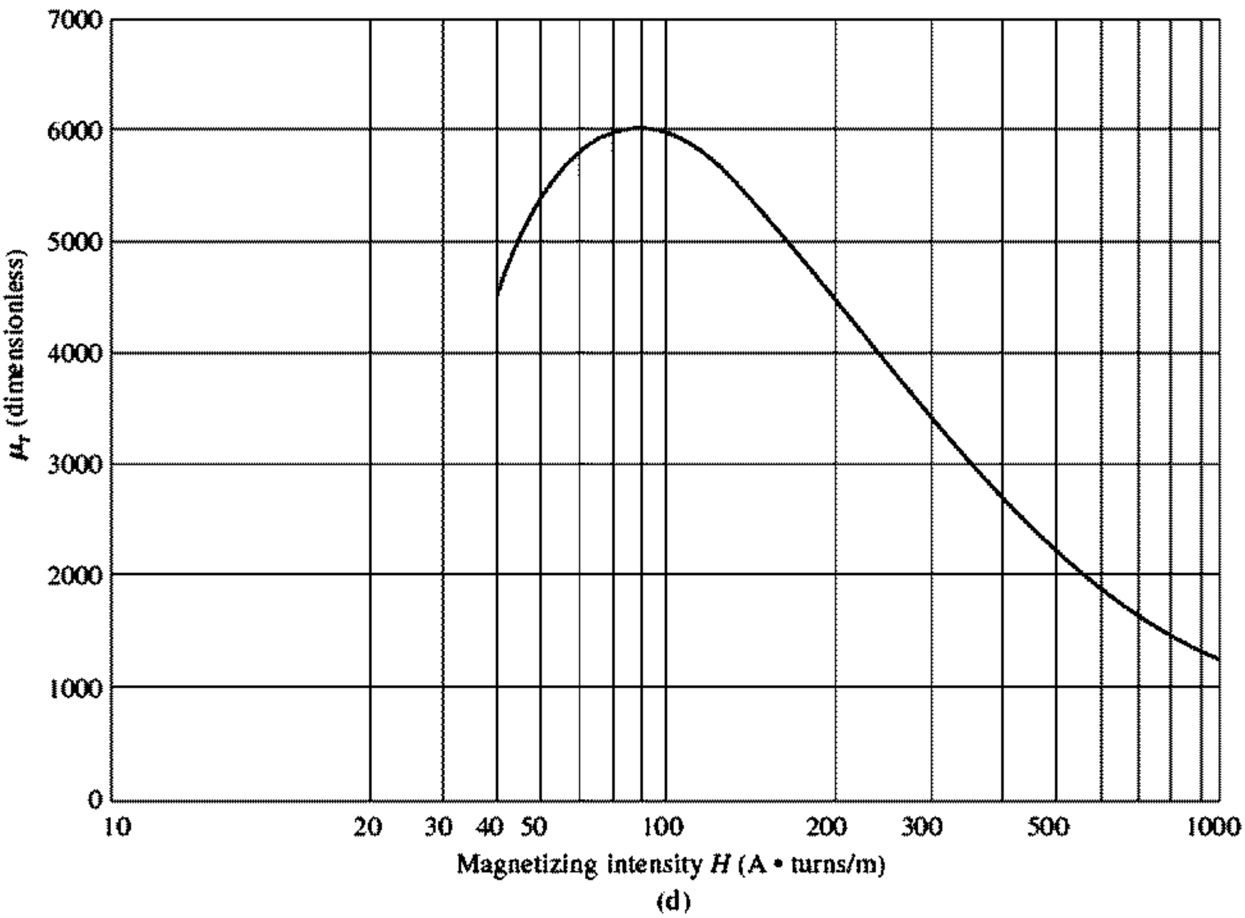


FIGURE 1–10
(continued)

between flux and magnetomotive force. The slope of the curve of flux density versus magnetizing intensity at any value of H in Figure 1–10b is by definition the permeability of the core at that magnetizing intensity. The curve shows that the permeability is large and relatively constant in the unsaturated region and then gradually drops to a very low value as the core becomes heavily saturated.

Figure 1–10c is a magnetization curve for a typical piece of steel shown in more detail and with the magnetizing intensity on a logarithmic scale. Only with the magnetizing intensity shown logarithmically can the huge saturation region of the curve fit onto the graph.

The advantage of using a ferromagnetic material for cores in electric machines and transformers is that one gets many times more flux for a given magnetomotive force with iron than with air. However, if the resulting flux has to be proportional, or nearly so, to the applied magnetomotive force, then the core *must* be operated in the unsaturated region of the magnetization curve.

Since real generators and motors depend on magnetic flux to produce voltage and torque, they are designed to produce as much flux as possible. As a result, most real machines operate near the knee of the magnetization curve, and the flux in their cores is not linearly related to the magnetomotive force producing it. This nonlinearity accounts for many of the peculiar behaviors of machines that will be explained in future chapters. We will use MATLAB to calculate solutions to problems involving the nonlinear behavior of real machines.

Example 1–4. Find the relative permeability of the typical ferromagnetic material whose magnetization curve is shown in Figure 1–10c at (a) $H = 50$, (b) $H = 100$, (c) $H = 500$, and (d) $H = 1000$ A • turns/m.

Solution

The permeability of a material is given by

$$\mu = \frac{B}{H}$$

and the relative permeability is given by

$$\mu_r = \frac{\mu}{\mu_0} \tag{1–23}$$

Thus, it is easy to determine the permeability at any given magnetizing intensity.

(a) At $H = 50$ A • turns/m, $B = 0.25$ T, so

$$\mu = \frac{B}{H} = \frac{0.25\ \text{T}}{50\ \text{A} \bullet \text{turns/m}} = 0.0050\ \text{H/m}$$

and

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.0050\ \text{H/m}}{4\pi \times 10^{-7}\ \text{H/m}} = 3980$$

(b) At $H = 100$ A • turns/m, $B = 0.72$ T, so

$$\mu = \frac{B}{H} = \frac{0.72\ \text{T}}{100\ \text{A} \bullet \text{turns/m}} = 0.0072\ \text{H/m}$$

and

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.0072\ \text{H/m}}{4\pi \times 10^{-7}\ \text{H/m}} = 5730$$

(c) At $H = 500$ A • turns/m, $B = 1.40$ T, so

$$\mu = \frac{B}{H} = \frac{1.40\ \text{T}}{500\ \text{A} \bullet \text{turns/m}} = 0.0028\ \text{H/m}$$

and

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.0028\ \text{H/m}}{4\pi \times 10^{-7}\ \text{H/m}} = 2230$$

(d) At $H = 1000$ A • turns/m, $B = 1.51$ T, so

$$\mu = \frac{B}{H} = \frac{1.51\ \text{T}}{1000\ \text{A} \bullet \text{turns/m}} = 0.00151\ \text{H/m}$$

and

$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.00151\ \text{H/m}}{4\pi \times 10^{-7}\ \text{H/m}} = 1200$$

Notice that as the magnetizing intensity is increased, the relative permeability first increases and then starts to drop off. The relative permeability of a typical ferromagnetic material as a function of the magnetizing intensity is shown in Figure 1–10d. This shape is fairly typical of all ferromagnetic materials. It can easily be seen from the curve for μ_r versus H that the assumption of constant relative permeability made in Examples 1–1 to 1–3 is valid only over a relatively narrow range of magnetizing intensities (or magnetomotive forces).

In the following example, the relative permeability is not assumed constant. Instead, the relationship between B and H is given by a graph.

Example 1–5. A square magnetic core has a mean path length of 55 cm and a cross-sectional area of 150 cm^2. A 200-turn coil of wire is wrapped around one leg of the core. The core is made of a material having the magnetization curve shown in Figure 1–10c.

(*a*) How much current is required to produce 0.012 Wb of flux in the core?
(*b*) What is the core's relative permeability at that current level?
(*c*) What is its reluctance?

Solution

(*a*) The required flux density in the core is

$$B = \frac{\phi}{A} = \frac{1.012 \text{ Wb}}{0.015 \text{ m}^2} = 0.8 \text{ T}$$

From Figure 1–10c, the required magnetizing intensity is

$$H = 115 \text{ A} \bullet \text{turns/m}$$

From Equation (1–20), the magnetomotive force needed to produce this magnetizing intensity is

$$\begin{aligned} \mathcal{F} &= Ni = Hl_c \\ &= (115 \text{ A} \bullet \text{turns/m})(0.55 \text{ m}) = 63.25 \text{ A} \bullet \text{turns} \end{aligned}$$

so the required current is

$$i = \frac{\mathcal{F}}{N} = \frac{63.25 \text{ A} \bullet \text{turns}}{200 \text{ turns}} = 0.316 \text{ A}$$

(*b*) The core's permeability at this current is

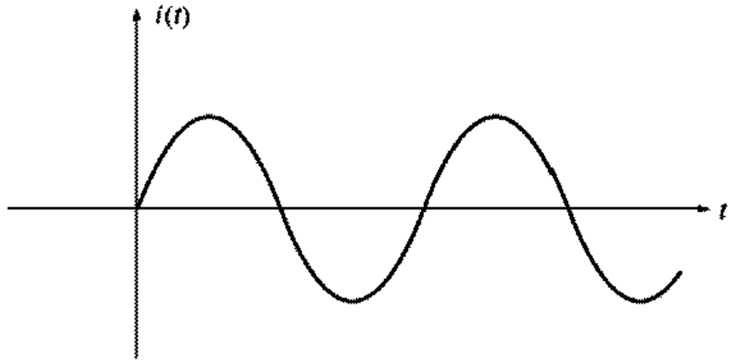
$$\mu = \frac{B}{H} = \frac{0.8 \text{ T}}{115 \text{ A} \bullet \text{turns/m}} = 0.00696 \text{ H/m}$$

Therefore, the relative permeability is

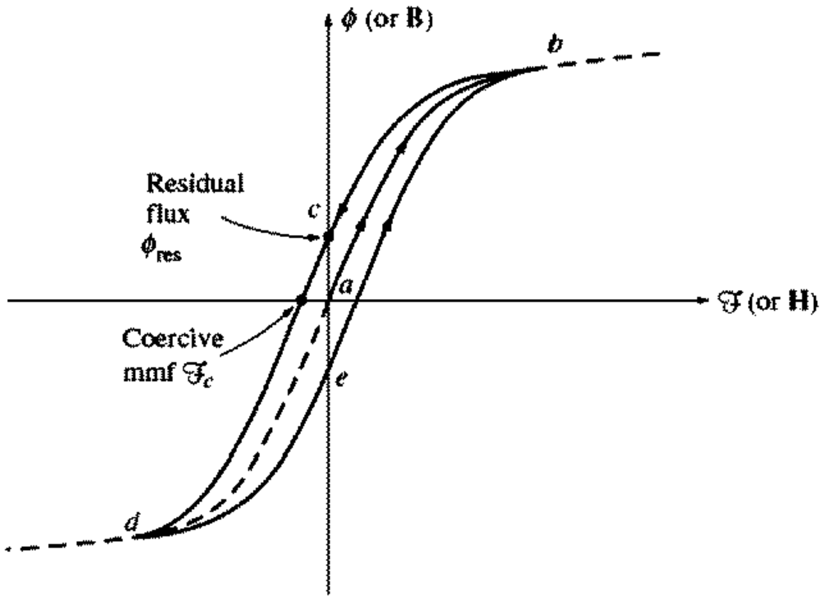
$$\mu_r = \frac{\mu}{\mu_0} = \frac{0.00696 \text{ H/m}}{4\pi \times 10^{-7} \text{ H/m}} = 5540$$

(*c*) The reluctance of the core is

$$\mathcal{R} = \frac{\mathcal{F}}{\phi} = \frac{63.25 \text{ A} \bullet \text{turns}}{0.012 \text{ Wb}} = 5270 \text{ A} \bullet \text{turns/Wb}$$



(a)



(b)

FIGURE 1–11
The hysteresis loop traced out by the flux in a core when the current $i(t)$ is applied to it.

Energy Losses in a Ferromagnetic Core

Instead of applying a direct current to the windings on the core, let us now apply an alternating current and observe what happens. The current to be applied is shown in Figure 1–11a. Assume that the flux in the core is initially zero. As the current increases for the first time, the flux in the core traces out path *ab* in Figure 1–11b. This is basically the saturation curve shown in Figure 1–10. However, when the current falls again, *the flux traces out a different path from the one it followed when the current increased.* As the current decreases, the flux in the core traces out path *bcd*, and later when the current increases again, the flux traces out path *deb*. Notice that the amount of flux present in the core depends not only on the amount of current applied to the windings of the core, but also on the previous history of the flux in the core. This dependence on the preceding flux history and the resulting failure to retrace flux paths is called *hysteresis*. Path *bcdeb* traced out in Figure 1–11b as the applied current changes is called a *hysteresis loop*.

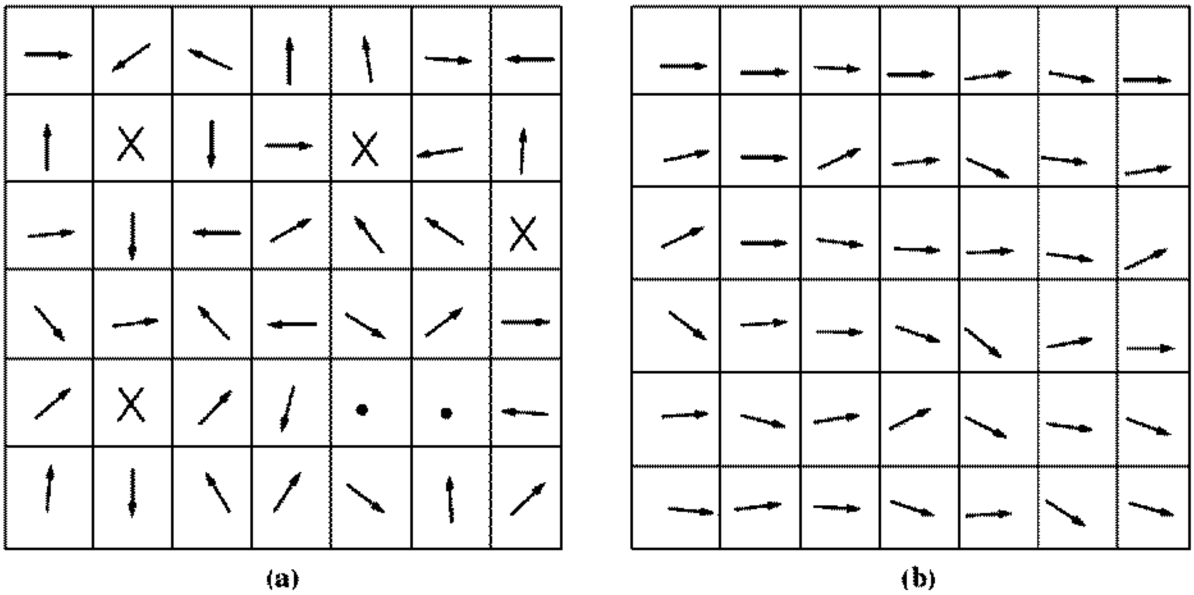


FIGURE 1–12
(a) Magnetic domains oriented randomly. (b) Magnetic domains lined up in the presence of an external magnetic field.

Notice that if a large magnetomotive force is first applied to the core and then removed, the flux path in the core will be *abc*. When the magnetomotive force is removed, the flux in the core ***does not*** go to zero. Instead, a magnetic field is left in the core. This magnetic field is called the ***residual flux*** in the core. It is in precisely this manner that permanent magnets are produced. To force the flux to zero, an amount of magnetomotive force known as the ***coercive magnetomotive force*** $\mathcal{F}_c$ must be applied to the core in the opposite direction.

Why does hysteresis occur? To understand the behavior of ferromagnetic materials, it is necessary to know something about their structure. The atoms of iron and similar metals (cobalt, nickel, and some of their alloys) tend to have their magnetic fields closely aligned with each other. Within the metal, there are many small regions called ***domains***. In each domain, all the atoms are aligned with their magnetic fields pointing in the same direction, so each domain within the material acts as a small permanent magnet. The reason that a whole block of iron can appear to have no flux is that these numerous tiny domains are oriented randomly within the material. An example of the domain structure within a piece of iron is shown in Figure 1–12.

When an external magnetic field is applied to this block of iron, it causes domains that happen to point in the direction of the field to grow at the expense of domains pointed in other directions. Domains pointing in the direction of the magnetic field grow because the atoms at their boundaries physically switch orientation to align themselves with the applied magnetic field. The extra atoms aligned with the field increase the magnetic flux in the iron, which in turn causes more atoms to switch orientation, further increasing the strength of the magnetic field. It is this positive feedback effect that causes iron to have a permeability much higher than air.

As the strength of the external magnetic field continues to increase, whole domains that are aligned in the wrong direction eventually reorient themselves as a unit to line up with the field. Finally, when nearly all the atoms and domains in the iron are lined up with the external field, any further increase in the magnetomotive force can cause only the same flux increase that it would in free space. (Once everything is aligned, there can be no more feedback effect to strengthen the field.) At this point, the iron is *saturated* with flux. This is the situation in the saturated region of the magnetization curve in Figure 1–10.

The key to hysteresis is that when the external magnetic field is removed, the domains do not completely randomize again. Why do the domains remain lined up? Because turning the atoms in them requires *energy*. Originally, energy was provided by the external magnetic field to accomplish the alignment; when the field is removed, there is no source of energy to cause all the domains to rotate back. The piece of iron is now a permanent magnet.

Once the domains are aligned, some of them will remain aligned until a source of external energy is supplied to change them. Examples of sources of external energy that can change the boundaries between domains and/or the alignment of domains are magnetomotive force applied in another direction, a large mechanical shock, and heating. Any of these events can impart energy to the domains and enable them to change alignment. (It is for this reason that a permanent magnet can lose its magnetism if it is dropped, hit with a hammer, or heated.)

The fact that turning domains in the iron requires energy leads to a common type of energy loss in all machines and transformers. The *hysteresis loss* in an iron core is the energy required to accomplish the reorientation of domains during each cycle of the alternating current applied to the core. It can be shown that the area enclosed in the hysteresis loop formed by applying an alternating current to the core is directly proportional to the energy lost in a given ac cycle. The smaller the applied magnetomotive force excursions on the core, the smaller the area of the resulting hysteresis loop and so the smaller the resulting losses. Figure 1–13 illustrates this point.

Another type of loss should be mentioned at this point, since it is also caused by varying magnetic fields in an iron core. This loss is the *eddy current* loss. The mechanism of eddy current losses is explained later after Faraday's law has been introduced. Both hysteresis and eddy current losses cause heating in the core material, and both losses must be considered in the design of any machine or transformer. Since both losses occur within the metal of the core, they are usually lumped together and called *core losses*.

1.5 FARADAY'S LAW—INDUCED VOLTAGE FROM A TIME-CHANGING MAGNETIC FIELD

So far, attention has been focused on the production of a magnetic field and on its properties. It is now time to examine the various ways in which an existing magnetic field can affect its surroundings.

The first major effect to be considered is called *Faraday's law*. It is the basis of transformer operation. Faraday's law states that if a flux passes through a

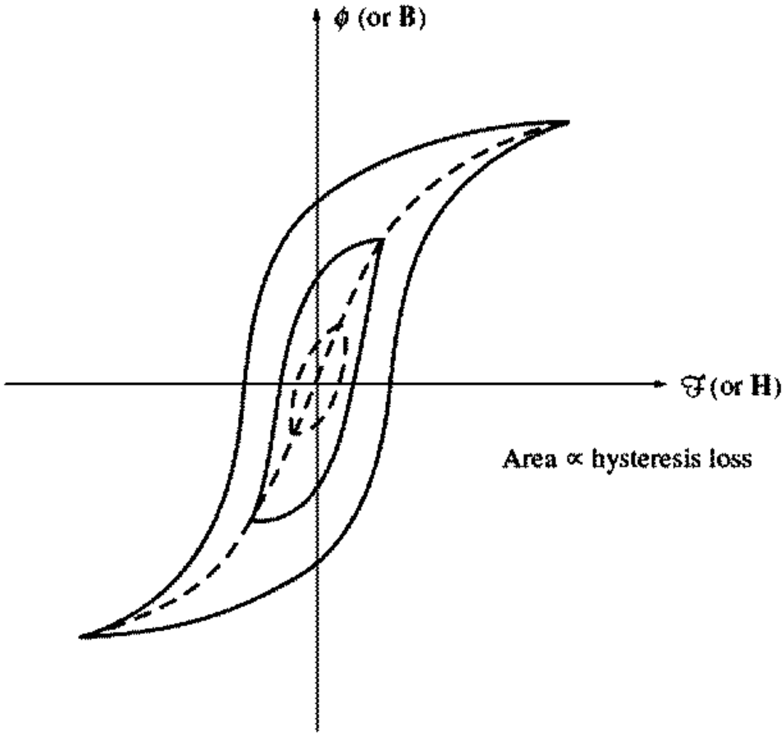


FIGURE 1–13
The effect of the size of magnetomotive force excursions on the magnitude of the hysteresis loss.

turn of a coil of wire, a voltage will be induced in the turn of wire that is directly proportional to the *rate of change* in the flux with respect to time. In equation form,

$$e_{\text{ind}} = -\frac{d\phi}{dt} \tag{1–35}$$

where e_{ind} is the voltage induced in the turn of the coil and ϕ is the flux passing through the turn. If a coil has N turns and if the same flux passes through all of them, then the voltage induced across the whole coil is given by

$$\boxed{e_{\text{ind}} = -N\frac{d\phi}{dt}} \tag{1–36}$$

where

e_{ind} = voltage induced in the coil
N = number of turns of wire in coil
ϕ = flux passing through coil

The minus sign in the equations is an expression of *Lenz's law.* Lenz's law states that the direction of the voltage buildup in the coil is such that if the coil ends were short circuited, it would produce current that would cause a flux *opposing* the original flux change. Since the induced voltage opposes the change that causes it, a minus sign is included in Equation (1–36). To understand this concept clearly,

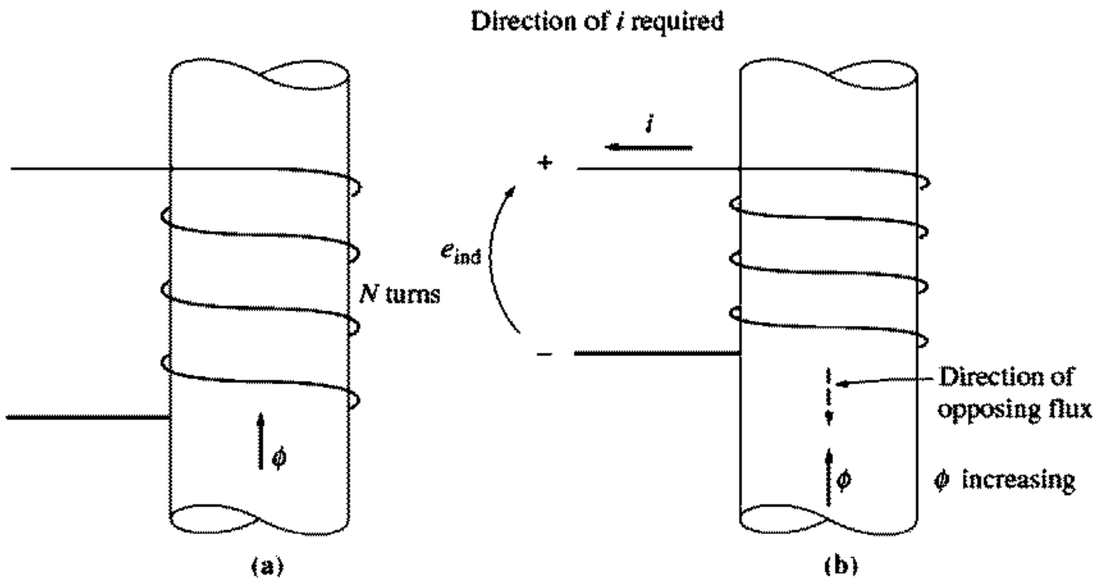


FIGURE 1–14
The meaning of Lenz's law: (a) A coil enclosing an increasing magnetic flux; (b) determining the resulting voltage polarity.

examine Figure 1–14. If the flux shown in the figure is *increasing* in strength, then the voltage built up in the coil will tend to establish a flux that will oppose the increase. A current flowing as shown in Figure 1–14b would produce a flux opposing the increase, so the voltage on the coil must be built up with the polarity required to drive that current through the external circuit. Therefore, the voltage must be built up with the polarity shown in the figure. Since the polarity of the resulting voltage can be determined from physical considerations, the minus sign in Equations (1–35) and (1–36) is often left out. It is left out of Faraday's law in the remainder of this book.

There is one major difficulty involved in using Equation (1–36) in practical problems. That equation assumes that exactly the same flux is present in each turn of the coil. Unfortunately, the flux leaking out of the core into the surrounding air prevents this from being true. If the windings are tightly coupled, so that the vast majority of the flux passing through one turn of the coil does indeed pass through all of them, then Equation (1–36) will give valid answers. But if leakage is quite high or if extreme accuracy is required, a different expression that does not make that assumption will be needed. The magnitude of the voltage in the ith turn of the coil is always given by

$$e_{ind} = \frac{d(\phi_i)}{dt} \tag{1–37}$$

If there are N turns in the coil of wire, the total voltage on the coil is

$$e_{ind} = \sum_{i=1}^{N} e_i \tag{1–38}$$

$$= \sum_{i=1}^{N} \frac{d(\phi_i)}{dt} \qquad (1\text{–}39)$$

$$= \frac{d}{dt}\left(\sum_{i=1}^{N} \phi_i\right) \qquad (1\text{–}40)$$

The term in parentheses in Equation (1–40) is called *the flux linkage* λ of the coil, and Faraday's law can be rewritten in terms of flux linkage as

$$\boxed{e_{\text{ind}} = \frac{d\lambda}{dt}} \qquad (1\text{–}41)$$

where

$$\boxed{\lambda = \sum_{i=1}^{N} \phi_i} \qquad (1\text{–}42)$$

The units of flux linkage are weber-turns.

Faraday's law is the fundamental property of magnetic fields involved in transformer operation. The effect of Lenz's law in transformers is to predict the polarity of the voltages induced in transformer windings.

Faraday's law also explains the eddy current losses mentioned previously. A time-changing flux induces voltage *within* a ferromagnetic core in just the same manner as it would in a wire wrapped around that core. These voltages cause swirls of current to flow within the core, much like the eddies seen at the edges of a river. It is the shape of these currents that gives rise to the name *eddy currents.* These eddy currents are flowing in a resistive material (the iron of the core), so energy is dissipated by them. The lost energy goes into heating the iron core.

The amount of energy lost to eddy currents is proportional to the size of the paths they follow within the core. For this reason, it is customary to break up any ferromagnetic core that may be subject to alternating fluxes into many small strips, or *laminations,* and to build the core up out of these strips. An insulating oxide or resin is used between the strips, so that the current paths for eddy currents are limited to very small areas. Because the insulating layers are extremely thin, this action reduces eddy current losses with very little effect on the core's magnetic properties. Actual eddy current losses are proportional to the square of the lamination thickness, so there is a strong incentive to make the laminations as thin as economically possible.

Example 1–6. Figure 1–15 shows a coil of wire wrapped around an iron core. If the flux in the core is given by the equation

$$\phi = 0.05 \sin 377t \qquad \text{Wb}$$

If there are 100 turns on the core, what voltage is produced at the terminals of the coil? Of what polarity is the voltage during the time when flux is *increasing* in the reference

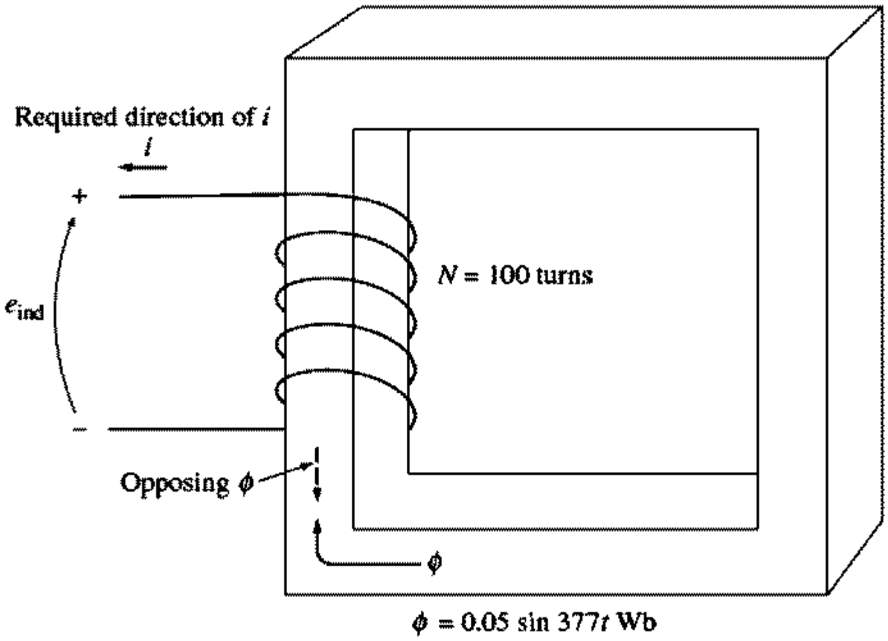


FIGURE 1–15
The core of Example 1–6. Determination of the voltage polarity at the terminals is shown.

direction shown in the figure? Assume that all the magnetic flux stays within the core (i.e., assume that the flux leakage is zero).

Solution

By the same reasoning as in the discussion on pages 29–30, the direction of the voltage while the flux is increasing in the reference direction must be positive to negative, as shown in Figure 1–15. The *magnitude* of the voltage is given by

$$\begin{aligned} e_{ind} &= N\frac{d\phi}{dt} \\ &= (100 \text{ turns})\frac{d}{dt}(0.05 \sin 377t) \\ &= 1885 \cos 377t \end{aligned}$$

or alternatively,

$$e_{ind} = 1885 \sin(377t + 90°) \text{ V}$$

1.6 PRODUCTION OF INDUCED FORCE ON A WIRE

A second major effect of a magnetic field on its surroundings is that it induces a force on a current-carrying wire within the field. The basic concept involved is illustrated in Figure 1–16. The figure shows a conductor present in a uniform magnetic field of flux density **B**, pointing into the page. The conductor itself is l meters long and contains a current of i amperes. The force induced on the conductor is given by

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) \tag{1–43}$$

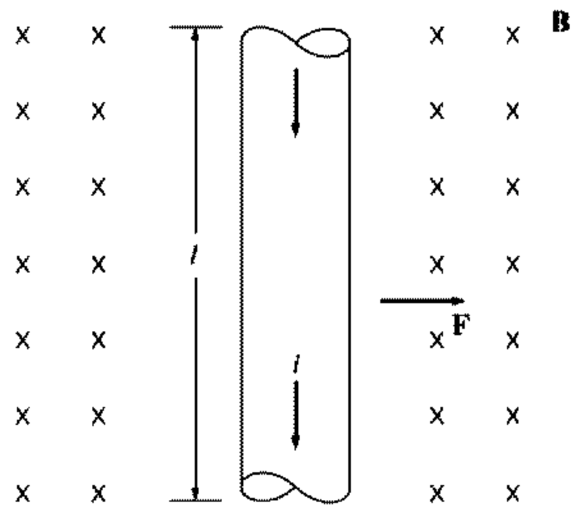


FIGURE 1–16
A current-carrying wire in the presence of a magnetic field.

where

i = magnitude of current in wire
$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow
$\mathbf{B}$ = magnetic flux density vector

The direction of the force is given by the right-hand rule: If the index finger of the right hand points in the direction of the vector $\mathbf{l}$ and the middle finger points in the direction of the flux density vector $\mathbf{B}$, then the thumb points in the direction of the resultant force on the wire. The magnitude of the force is given by the equation

$$F = ilB \sin \theta \tag{1–44}$$

where θ is the angle between the wire and the flux density vector.

Example 1–7. Figure 1–16 shows a wire carrying a current in the presence of a magnetic field. The magnetic flux density is 0.25 T, directed into the page. If the wire is 1.0 m long and carries 0.5 A of current in the direction from the top of the page to the bottom of the page, what are the magnitude and direction of the force induced on the wire?

Solution
The direction of the force is given by the right-hand rule as being to the right. The magnitude is given by

$$\begin{aligned} F &= ilB \sin \theta \qquad (1\text{–}44) \\ &= (0.5\ \text{A})(1.0\ \text{m})(0.25\ \text{T}) \sin 90^\circ = 0.125\ \text{N} \end{aligned}$$

Therefore,

$$\mathbf{F} = 0.125\ \text{N, directed to the right}$$

The induction of a force in a wire by a current in the presence of a magnetic field is the basis of *motor action*. Almost every type of motor depends on this basic principle for the forces and torques which make it move.

1.7 INDUCED VOLTAGE ON A CONDUCTOR MOVING IN A MAGNETIC FIELD

There is a third major way in which a magnetic field interacts with its surroundings. If a wire with the proper orientation moves through a magnetic field, a voltage is induced in it. This idea is shown in Figure 1–17. The voltage induced in the wire is given by

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \tag{1–45}$$

where

$\mathbf{v}$ = velocity of the wire

$\mathbf{B}$ = magnetic flux density vector

$\mathbf{l}$ = length of conductor in the magnetic field

Vector $\mathbf{l}$ points along the direction of the wire toward the end making the smallest angle with respect to the vector $\mathbf{v} \times \mathbf{B}$. The voltage in the wire will be built up so that the positive end is in the direction of the vector $\mathbf{v} \times \mathbf{B}$. The following examples illustrate this concept.

Example 1–8. Figure 1–17 shows a conductor moving with a velocity of 5.0 m/s to the right in the presence of a magnetic field. The flux density is 0.5 T into the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

Solution

The direction of the quantity $\mathbf{v} \times \mathbf{B}$ in this example is up. Therefore, the voltage on the conductor will be built up positive at the top with respect to the bottom of the wire. The direction of vector $\mathbf{l}$ is up, so that it makes the smallest angle with respect to the vector $\mathbf{v} \times \mathbf{B}$.

Since $\mathbf{v}$ is perpendicular to $\mathbf{B}$ and since $\mathbf{v} \times \mathbf{B}$ is parallel to $\mathbf{l}$, the magnitude of the induced voltage reduces to

$$\begin{aligned} e_{\text{ind}} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45) \\ &= (vB \sin 90^\circ)\, l \cos 0^\circ \\ &= vBl \\ &= (5.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \\ &= 2.5 \text{ V} \end{aligned}$$

Thus the induced voltage is 2.5 V, positive at the top of the wire.

Example 1–9. Figure 1–18 shows a conductor moving with a velocity of 10 m/s to the right in a magnetic field. The flux density is 0.5 T, out of the page, and the wire is 1.0 m in length, oriented as shown. What are the magnitude and polarity of the resulting induced voltage?

Solution

The direction of the quantity $\mathbf{v} \times \mathbf{B}$ is down. The wire is not oriented on an up-down line, so choose the direction of $\mathbf{l}$ as shown to make the smallest possible angle with the direction

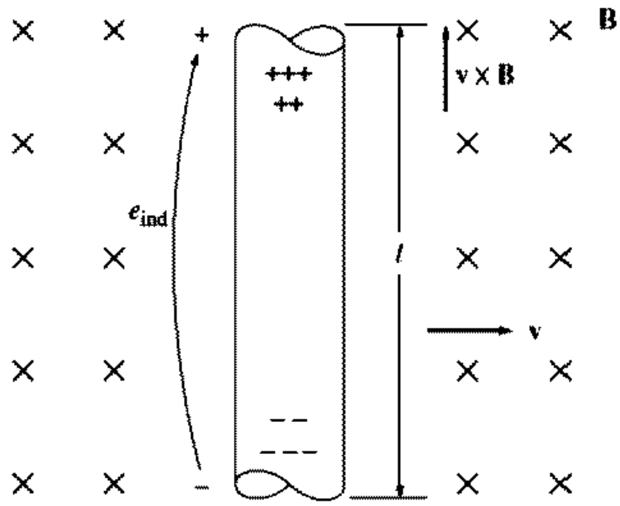


FIGURE 1–17
A conductor moving in the presence of a magnetic field.

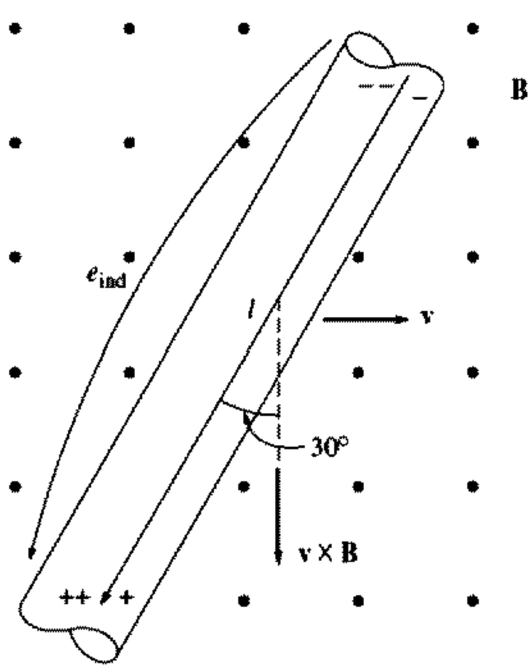


FIGURE 1–18
The conductor of Example 1–9.

of $\mathbf{v} \times \mathbf{B}$. The voltage is positive at the bottom of the wire with respect to the top of the wire. The magnitude of the voltage is

$$\begin{aligned} e_{ind} &= (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} \qquad (1\text{–}45) \\ &= (vB \sin 90°)\, l \cos 30° \\ &= (10.0 \text{ m/s})(0.5 \text{ T})(1.0 \text{ m}) \cos 30° \\ &= 4.33 \text{ V} \end{aligned}$$

The induction of voltages in a wire moving in a magnetic field is fundamental to the operation of all types of generators. For this reason, it is called *generator action.*

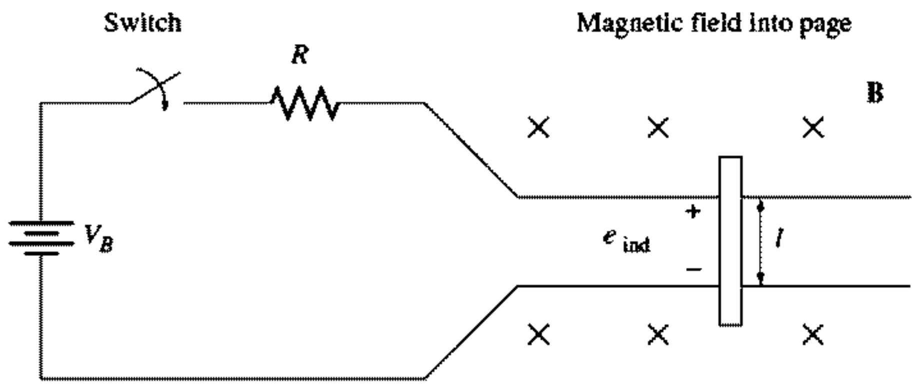


FIGURE 1–19
A linear dc machine. The magnetic field points into the page.

1.8 THE LINEAR DC MACHINE—A SIMPLE EXAMPLE

A *linear dc machine* is about the simplest and easiest-to-understand version of a dc machine, yet it operates according to the same principles and exhibits the same behavior as real generators and motors. It thus serves as a good starting point in the study of machines.

A linear dc machine is shown in Figure 1–19. It consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. Along the bed of this "railroad track" is a constant, uniform-density magnetic field directed into the page. A bar of conducting metal is lying across the tracks.

How does such a strange device behave? Its behavior can be determined from an application of four basic equations to the machine. These equations are

1. The equation for the force on a wire in the presence of a magnetic field:

$$\boxed{\mathbf{F} = i(\mathbf{l} \times \mathbf{B})} \tag{1–43}$$

where $\mathbf{F}$ = force on wire

i = magnitude of current in wire

$\mathbf{l}$ = length of wire, with direction of $\mathbf{l}$ defined to be in the direction of current flow

$\mathbf{B}$ = magnetic flux density vector

2. The equation for the voltage induced on a wire moving in a magnetic field:

$$\boxed{e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l}} \tag{1–45}$$

where e_{ind} = voltage induced in wire

$\mathbf{v}$ = velocity of the wire

$\mathbf{B}$ = magnetic flux density vector

$\mathbf{l}$ = length of conductor in the magnetic field

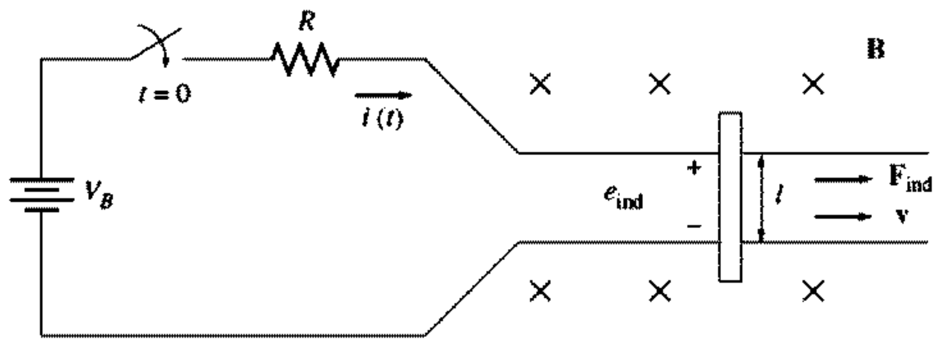


FIGURE 1–20
Starting a linear dc machine.

3. Kirchhoff's voltage law for this machine. From Figure 1–19 this law gives

$$V_B - iR - e_{ind} = 0$$

$$\boxed{V_B = e_{ind} + iR = 0} \tag{1–46}$$

4. Newton's law for the bar across the tracks:

$$\boxed{F_{net} = ma} \tag{1–7}$$

We will now explore the fundamental behavior of this simple dc machine using these four equations as tools.

Starting the Linear DC Machine

Figure 1–20 shows the linear dc machine under starting conditions. To start this machine, simply close the switch. Now a current flows in the bar, which is given by Kirchhoff's voltage law:

$$i = \frac{V_B - e_{ind}}{R} \tag{1–47}$$

Since the bar is initially at rest, $e_{ind} = 0$, so $i = V_B/R$. The current flows down through the bar across the tracks. But from Equation (1–43), a current flowing through a wire in the presence of a magnetic field induces a force on the wire. Because of the geometry of the machine, this force is

$$F_{ind} = ilB \qquad \text{to the right} \tag{1–48}$$

Therefore, the bar will accelerate to the right (by Newton's law). However, when the velocity of the bar begins to increase, a voltage appears across the bar. The voltage is given by Equation (1–45), which reduces for this geometry to

$$e_{ind} = vBl \qquad \text{positive upward} \tag{1–49}$$

The voltage now reduces the current flowing in the bar, since by Kirchhoff's voltage law

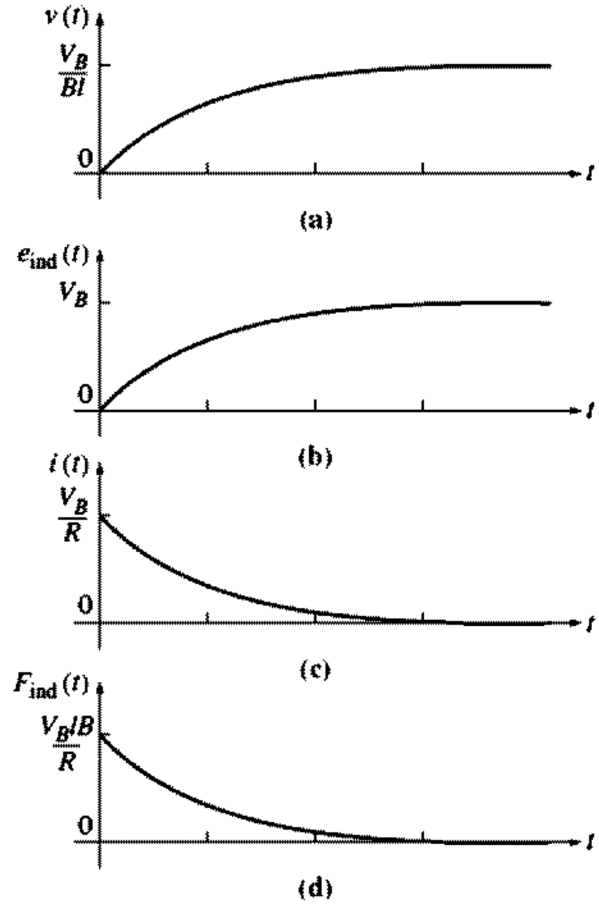


FIGURE 1–21
The linear dc machine on starting.
(a) Velocity $v(t)$ as a function of time;
(b) induced voltage $e_{\text{ind}}(t)$; (c) current $i(t)$;
(d) induced force $F_{\text{ind}}(t)$.

$$i\downarrow = \frac{V_B - e_{\text{ind}}\uparrow}{R} \tag{1–47}$$

As e_{ind} increases, the current i decreases.

The result of this action is that eventually the bar will reach a constant steady-state speed where the net force on the bar is zero. This will occur when e_{ind} has risen all the way up to equal the voltage V_B. At that time, the bar will be moving at a speed given by

$$V_B = e_{\text{ind}} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl} \tag{1–50}$$

The bar will continue to coast along at this no-load speed forever unless some external force disturbs it. When the motor is started, the velocity v, induced voltage e_{ind}, current i, and induced force F_{ind} are as sketched in Figure 1–21.

To summarize, at starting, the linear dc machine behaves as follows:

1. Closing the switch produces a current flow $i = V_B/R$.
2. The current flow produces a force on the bar given by $F = ilB$.

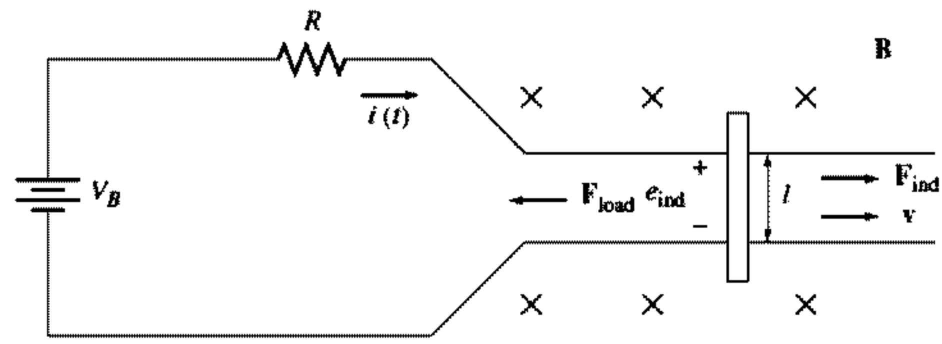


FIGURE 1–22
The linear dc machine as a motor.

3. The bar accelerates to the right, producing an induced voltage e_{ind} as it speeds up.
4. This induced voltage reduces the current flow $i = (V_B - e_{ind}\uparrow)/R$.
5. The induced force is thus decreased ($F = i\downarrow lB$) until eventually $F = 0$. At that point, $e_{ind} = V_B$, $i = 0$, and the bar moves at a constant no-load speed $v_{ss} = V_B/Bl$.

This is precisely the behavior observed in real motors on starting.

The Linear DC Machine as a Motor

Assume that the linear machine is initially running at the no-load steady-state conditions described above. What will happen to this machine if an external load is applied to it? To find out, let's examine Figure 1–22. Here, a force $\mathbf{F}_{load}$ is applied to the bar opposite the direction of motion. Since the bar was initially at steady state, application of the force $\mathbf{F}_{load}$ will result in a net force on the bar in the direction *opposite* the direction of motion ($\mathbf{F}_{net} = \mathbf{F}_{load} - \mathbf{F}_{ind}$). The effect of this force will be to slow the bar. But just as soon as the bar begins to slow down, the induced voltage on the bar drops ($e_{ind} = v\downarrow Bl$). As the induced voltage decreases, the current flow in the bar rises:

$$i\uparrow = \frac{V_B - e_{ind}\downarrow}{R} \tag{1–47}$$

Therefore, the induced force rises too ($F_{ind} = i\uparrow lB$). The overall result of this chain of events is that the induced force rises until it is equal and opposite to the load force, and the bar again travels in steady state, but at a lower speed. When a load is attached to the bar, the velocity v, induced voltage e_{ind}, current i, and induced force F_{ind} are as sketched in Figure 1–23.

There is now an induced force in the direction of motion of the bar, and power is being *converted from electrical form to mechanical form* to keep the bar moving. The power being converted is

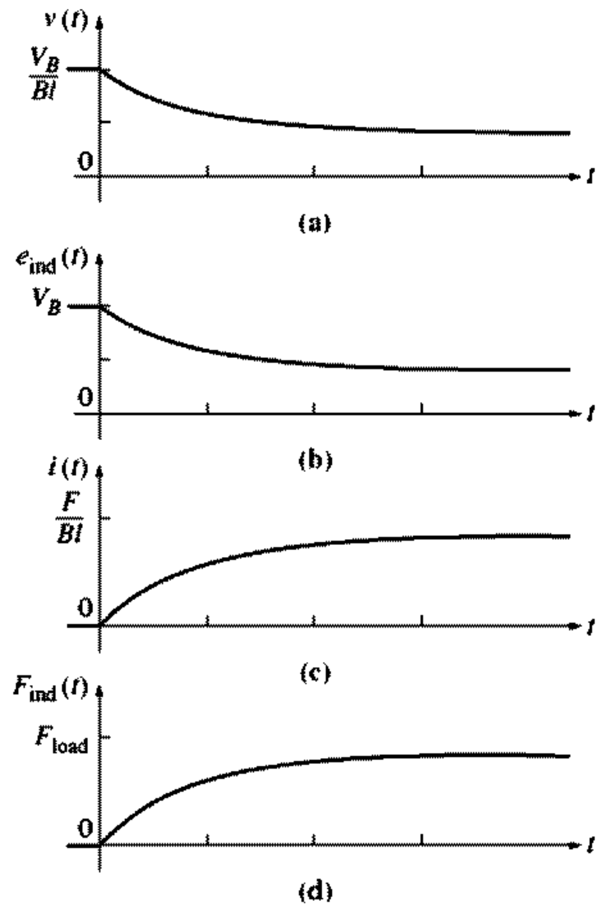


FIGURE 1–23
The linear dc machine operating at no-load conditions and then loaded as a motor.
(a) Velocity $v(t)$ as a function of time;
(b) induced voltage $e_{ind}(t)$; (c) current $i(t)$;
(d) induced force $F_{ind}(t)$.

$$P_{conv} = e_{ind}i = F_{ind}v \tag{1–51}$$

An amount of electric power equal to $e_{ind}i$ is consumed in the bar and is replaced by mechanical power equal to $F_{ind}v$. Since power is converted from electrical to mechanical form, this bar is operating as a *motor*.

To summarize this behavior:

1. A force $\mathbf{F}_{load}$ is applied opposite to the direction of motion, which causes a net force $\mathbf{F}_{net}$ opposite to the direction of motion.
2. The resulting acceleration $a = F_{net}/m$ is negative, so the bar slows down ($v\downarrow$).
3. The voltage $e_{ind} = v\downarrow Bl$ falls, and so $i = (V_B - e_{ind}\downarrow)/R$ increases.
4. The induced force $F_{ind} = i\uparrow lB$ increases until $|\mathbf{F}_{ind}| = |\mathbf{F}_{load}|$ at a lower speed v.
5. An amount of electric power equal to $e_{ind}i$ is now being converted to mechanical power equal to $F_{ind}v$, and the machine is acting as a motor.

A real dc motor behaves in a precisely analogous fashion when it is loaded: As a load is added to its shaft, the motor begins to slow down, which reduces its internal voltage, increasing its current flow. The increased current flow increases its induced torque, and the induced torque will equal the load torque of the motor at a new, slower speed.

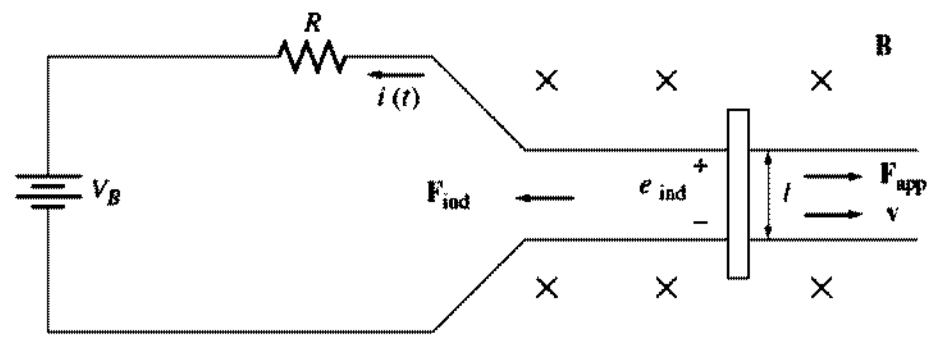


FIGURE 1–24
The linear dc machine as a generator.

Note that the power converted from electrical form to mechanical form by this linear motor was given by the equation $P_{conv} = F_{ind}v$. The power converted from electrical form to mechanical form in a real rotating motor is given by the equation

$$P_{conv} = \tau_{ind}\omega \tag{1–52}$$

where the induced torque τ_{ind} is the rotational analog of the induced force F_{ind}, and the angular velocity ω is the rotational analog of the linear velocity v.

The Linear DC Machine as a Generator

Suppose that the linear machine is again operating under no-load steady-state conditions. This time, apply a force *in the direction of motion* and see what happens.

Figure 1–24 shows the linear machine with an applied force $\mathbf{F}_{app}$ in the direction of motion. Now the applied force will cause the bar to accelerate in the direction of motion, and the velocity v of the bar will increase. As the velocity increases, $e_{ind} = v\uparrow Bl$ will increase and will be larger than the battery voltage V_B. With $e_{ind} > V_B$, the current reverses direction and is now given by the equation

$$i = \frac{e_{ind} - V_B}{R} \tag{1–53}$$

Since this current now flows *up* through the bar, it induces a force in the bar given by

$$F_{ind} = ilB \qquad \text{to the left} \tag{1–54}$$

The direction of the induced force is given by the right-hand rule. This induced force opposes the applied force on the bar.

Finally, the induced force will be equal and opposite to the applied force, and the bar will be moving at a *higher* speed than before. Notice that now *the battery is charging.* The linear machine is now serving as a generator, converting mechanical power $F_{ind}v$ into electric power $e_{ind}i$.

To summarize this behavior:

1. A force $\mathbf{F}_{\text{app}}$ is applied in the direction of motion; $\mathbf{F}_{\text{net}}$ is in the direction of motion.
2. Acceleration $a = F_{\text{net}}/m$ is positive, so the bar speeds up ($v\uparrow$).
3. The voltage $e_{\text{ind}} = v\uparrow Bl$ increases, and so $i = (e_{\text{ind}}\uparrow - V_B)/R$ increases.
4. The induced force $F_{\text{ind}} = i\uparrow lB$ increases until $|\mathbf{F}_{\text{ind}}| = |\mathbf{F}_{\text{load}}|$ at a higher speed v.
5. An amount of mechanical power equal to $F_{\text{ind}}v$ is now being converted to electric power $e_{\text{ind}}i$, and the machine is acting as a generator.

Again, a real dc generator behaves in precisely this manner: A torque is applied to the shaft *in the direction of motion,* the speed of the shaft increases, the internal voltage increases, and current flows out of the generator to the loads. The amount of mechanical power converted to electrical form in the real rotating generator is again given by Equation (1–52):

$$P_{\text{conv}} = \tau_{\text{ind}}\omega \tag{1–52}$$

It is interesting that the same machine acts as *both motor and generator.* The only difference between the two is whether the externally applied forces are in the direction of motion (generator) or opposite to the direction of motion (motor). Electrically, when $e_{\text{ind}} > V_B$, the machine acts as a generator, and when $e_{\text{ind}} < V_B$, the machine acts as a motor. Whether the machine is a motor or a generator, both induced force (motor action) and induced voltage (generator action) are present at all times. This is generally true of all machines—both actions are present, and it is only the relative directions of the external forces with respect to the direction of motion that determine whether the overall machine behaves as a motor or as a generator.

Another very interesting fact should be noted: This machine was a generator when it moved rapidly and a motor when it moved more slowly, but whether it was a motor or a generator, it always moved in the same direction. Many beginning machinery students expect a machine to turn one way as a generator and the other way as a motor. *This does not occur.* Instead, there is merely a small change in operating speed and a reversal of current flow.

Starting Problems with the Linear Machine

A linear machine is shown in Figure 1–25. This machine is supplied by a 250-V dc source, and its internal resistance R is given as about 0.10 Ω. (The resistor R models the internal resistance of a real dc machine, and this is a fairly reasonable internal resistance for a medium-size dc motor.)

Providing actual numbers in this figure highlights a major problem with machines (and their simple linear model). At starting conditions, the speed of the bar is zero, so $e_{\text{ind}} = 0$. The current flow at starting is

$$i_{start} = \frac{V_B}{R} = \frac{250\ V}{0.1\ \Omega} = 2500\ \text{A}$$

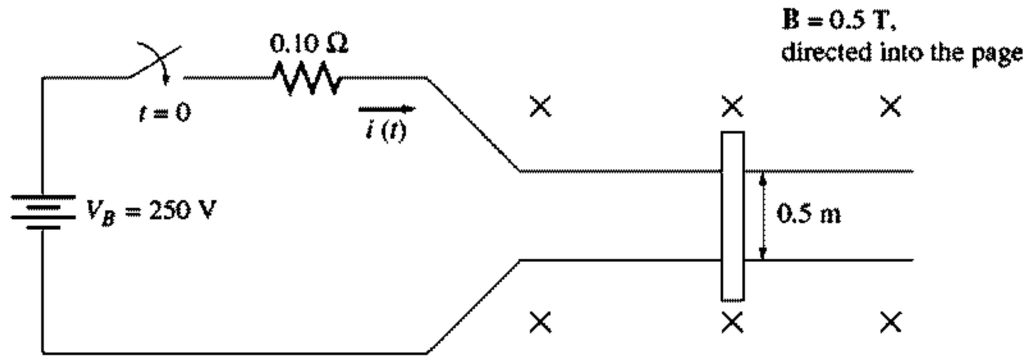


FIGURE 1–25
The linear dc machine with component values illustrating the problem of excessive starting current.

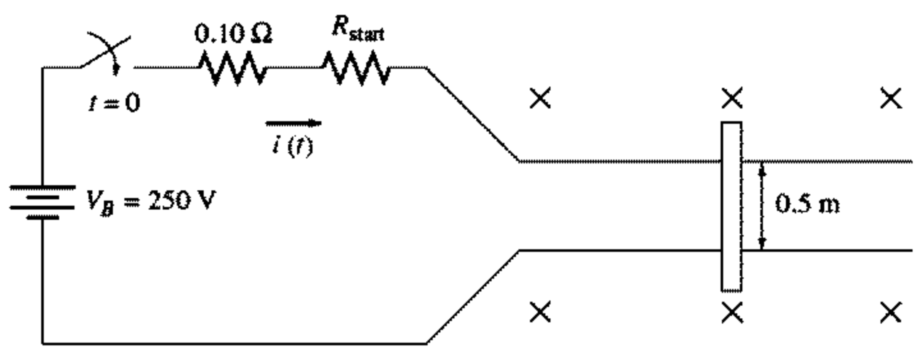


FIGURE 1–26
A linear dc machine with an extra series resistor inserted to control the starting current.

This current is very high, often in excess of 10 times the rated current of the machine. Such currents can cause severe damage to a motor. Both real ac and real dc machines suffer from similar high-current problems on starting.

How can such damage be prevented? The easiest method for this simple linear machine is to insert an extra resistance into the circuit during starting to limit the current flow until e_{ind} builds up enough to limit it. Figure 1–26 shows a starting resistance inserted into the machine circuitry.

The same problem exists in real dc machines, and it is handled in precisely the same fashion—a resistor is inserted into the motor armature circuit during starting. The control of high starting current in real ac machines is handled in a different fashion, which will be described in Chapter 8.

Example 1–10. The linear dc machine shown in Figure 1–27a has a battery voltage of 120 V, an internal resistance of 0.3 Ω, and a magnetic flux density of 0.1 T.

(a) What is this machine's maximum starting current? What is its steady-state velocity at no load?

(b) Suppose that a 30-N force pointing to the right were applied to the bar. What would the steady-state speed be? How much power would the bar be producing or consuming? How much power would the battery be producing or consuming?

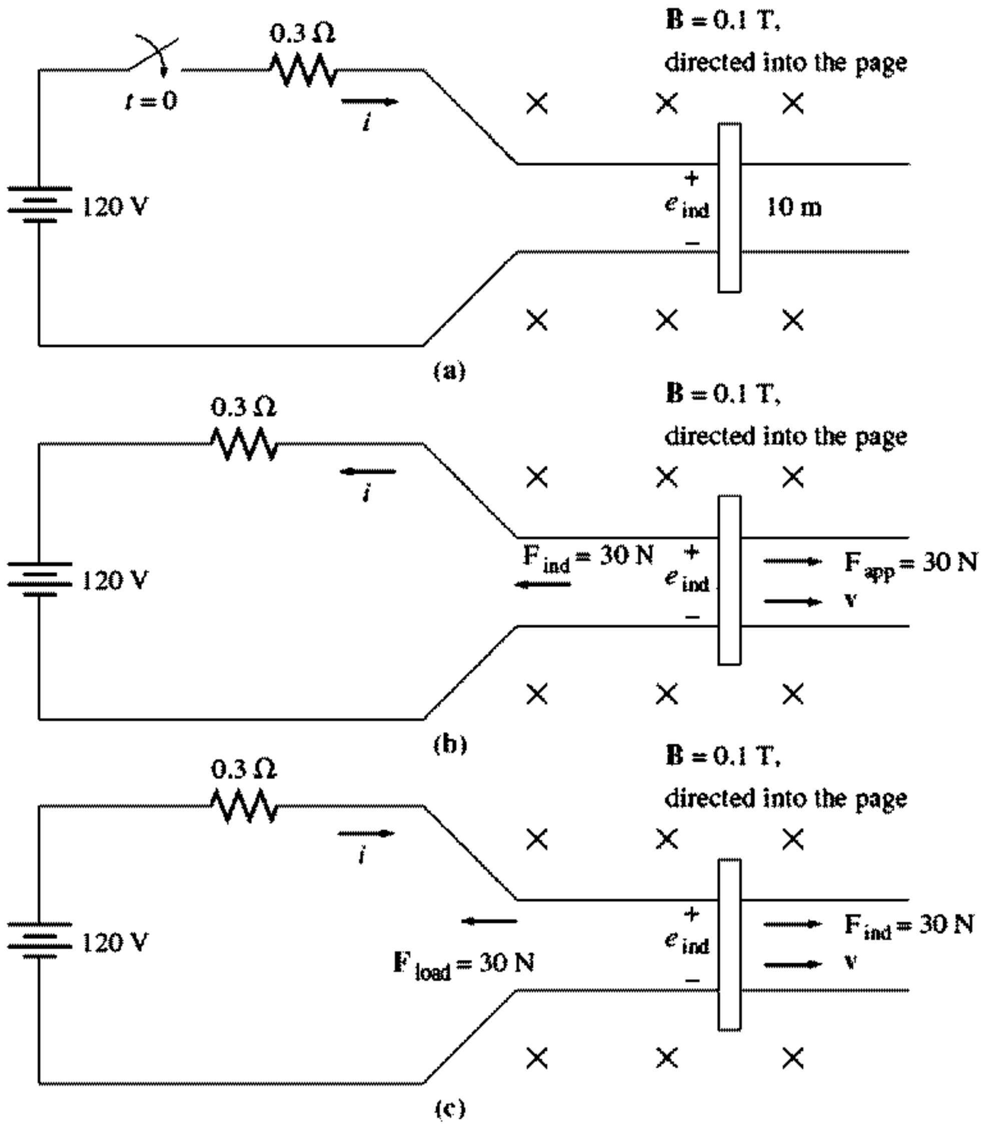


FIGURE 1–27
The linear dc machine of Example 1–10. (a) Starting conditions; (b) operating as a generator; (c) operating as a motor.

Explain the difference between these two figures. Is this machine acting as a motor or as a generator?

(c) Now suppose a 30-N force pointing to the left were applied to the bar. What would the new steady-state speed be? Is this machine a motor or a generator now?

(d) Assume that a force pointing to the left is applied to the bar. Calculate speed of the bar as a function of the force for values from 0 N to 50 N in 10-N steps. Plot the velocity of the bar versus the applied force.

(e) Assume that the bar is unloaded and that it suddenly runs into a region where the magnetic field is weakened to 0.08 T. How fast will the bar go now?

Solution

(a) At starting conditions, the velocity of the bar is 0, so $e_{ind} = 0$. Therefore,

$$i = \frac{V_B - e_{ind}}{R} = \frac{120\text{ V} - 0\text{ V}}{0.3\ \Omega} = 400\text{ A}$$

When the machine reaches steady state, $\mathbf{F}_{ind} = 0$ and $i = 0$. Therefore,

$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 120 \text{ m/s}$$

(b) Refer to Figure 1–27b. If a 30-N force to the right is applied to the bar, the final steady state will occur when the induced force $\mathbf{F}_{ind}$ is equal and opposite to the applied force $\mathbf{F}_{app}$, so that the net force on the bar is zero:

$$F_{app} = F_{ind} = ilB$$

Therefore,

$$i = \frac{F_{ind}}{lB} = \frac{30 \text{ N}}{(10\text{m})(0.1 \text{ T})}$$

$$= 30 \text{ A} \quad \text{flowing up through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B + iR$$

$$= 120 \text{ V} + (30\text{A})(0.3\ \Omega) = 129 \text{ V}$$

and the final steady-state speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{129 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 129 \text{ m/s}$$

The bar is *producing* $P = (129 \text{ V})(30 \text{ A}) = 3870$ W of power, and the battery is *consuming* $P = (120 \text{ V})(30 \text{ A}) = 3600$ W. The difference between these two numbers is the 270 W of losses in the resistor. This machine is acting as a *generator.*

(c) Refer to Figure 1–25c. This time, the force is applied to the left, and the induced force is to the right. At steady state,

$$F_{app} = F_{ind} = ilB$$

$$i = \frac{F_{ind}}{lB} = \frac{30 \text{ N}}{(10 \text{ m})(0.1 \text{ T})}$$

$$= 30 \text{ A} \quad \text{flowing down through the bar}$$

The induced voltage e_{ind} on the bar must be

$$e_{ind} = V_B - iR$$

$$= 120 \text{ V} - (30 \text{ A})(0.3\ \Omega) = 111 \text{ V}$$

and the final speed must be

$$v_{ss} = \frac{e_{ind}}{Bl}$$

$$= \frac{111 \text{ V}}{(0.1 \text{ T})(10 \text{ m})} = 111 \text{ m/s}$$

This machine is now acting as a *motor,* converting electric energy from the battery into mechanical energy of motion on the bar.

(d) This task is ideally suited for MATLAB. We can take advantage of MATLAB's vectorized calculations to determine the velocity of the bar for each value of force. The MATLAB code to perform this calculation is just a version of the steps that were performed by hand in part *c*. The program shown below calculates the current, induced voltage, and velocity in that order, and then plots the velocity versus the force on the bar.

```
% M-file: ex1_10.m
% M-file to calculate and plot the velocity of
% a linear motor as a function of load.
VB = 120;                         % Battery voltage (V)
r = 0.3;                          % Resistance (ohms)
l = 1;                            % Bar length (m)
B = 0.6;                          % Flux density (T)

% Select the forces to apply to the bar
F = 0:10:50;                      % Force (N)

% Calculate the currents flowing in the motor.
i = F ./ (l * B);                 % Current (A)

% Calculate the induced voltages on the bar.
eind = VB - i .* r;               % Induced voltage (V)

% Calculate the velocities of the bar.
v_bar = eind ./ (l * B);          % Velocity (m/s)

% Plot the velocity of the bar versus force.
plot(F,v_bar);
title ('Plot of Velocity versus Applied Force');
xlabel ('Force (N)');
ylabel ('Velocity (m/s)');
axis ([0 50 0 200]);
```

The resulting plot is shown in Figure 1–28. Note that the bar slows down more and more as load increases.

(e) If the bar is initially unloaded, then $e_{ind} = V_B$. If the bar suddenly hits a region of weaker magnetic field, a transient will occur. Once the transient is over, though, e_{ind} will again equal V_B.

This fact can be used to determine the final speed of the bar. The *initial speed* was 120 m/s. The *final speed* is

$$VB = e_{ind} = v_{ss}Bl$$

$$v_{ss} = \frac{V_B}{Bl}$$

$$= \frac{120\text{ V}}{(0.08\text{ T})(10\text{ m})} = 150\text{ m/s}$$

Thus, when the flux in the linear motor weakens, the bar speeds up. The same behavior occurs in real dc motors: When the field flux of a dc motor weakens, it turns faster. Here, again, the linear machine behaves in much the same way as a real dc motor.

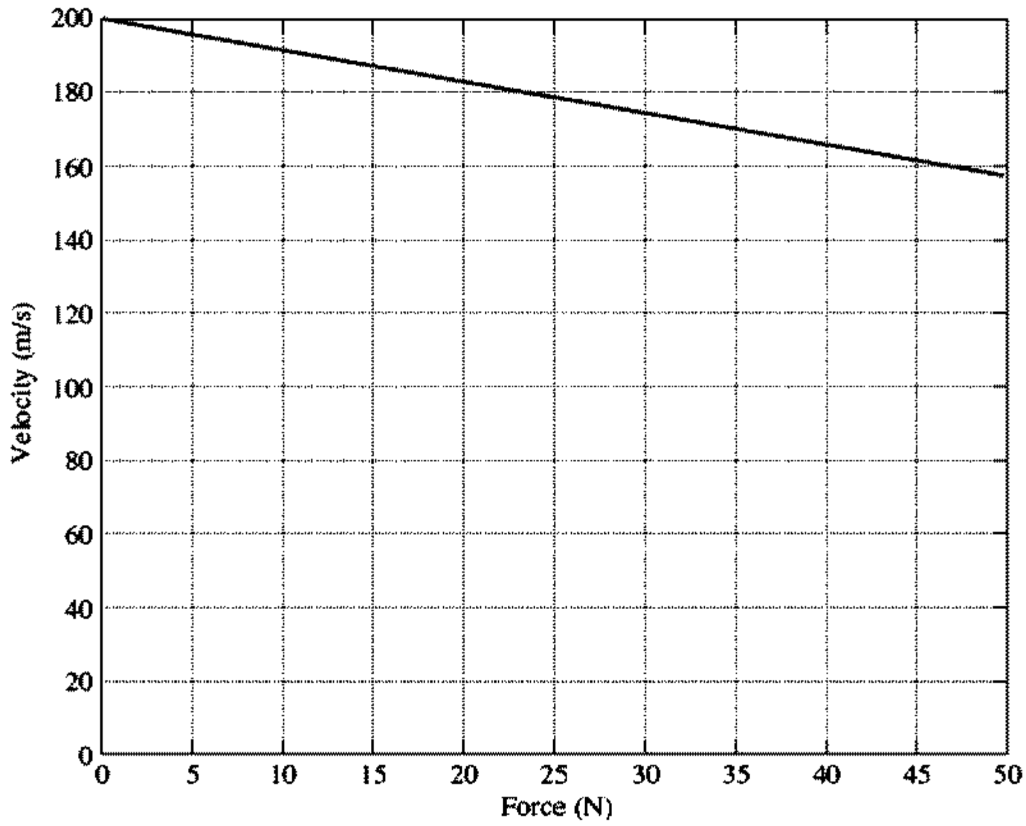


FIGURE 1–28
Plot of velocity versus force for a linear dc machine.

1.9 REAL, REACTIVE, AND APPARENT POWER IN AC CIRCUITS

In a dc circuit such as the one shown in Figure 1–29a, the power supplied to the dc load is simply the product of the voltage across the load and the current flowing through it.

$$P = VI \tag{1–55}$$

Unfortunately, the situation in sinusoidal ac circuits is more complex, because there can be a phase difference between the ac voltage and the ac current supplied to the load. The *instantaneous* power supplied to an ac load will still be the product of the instantaneous voltage and the instantaneous current, but the *average* power supplied to the load will be affected by the phase angle between the voltage and the current. We will now explore the effects of this phase difference on the average power supplied to an ac load.

Figure 1–29b shows a single-phase voltage source supplying power to a single-phase load with impedance $\mathbf{Z} = Z\angle\theta\ \Omega$. If we assume that the load is inductive, then the impedance angle θ of the load will be positive, and the current will lag the voltage by θ degrees.

The voltage applied to this load is

$$v(t) = \sqrt{2}V \cos \omega t \tag{1–56}$$

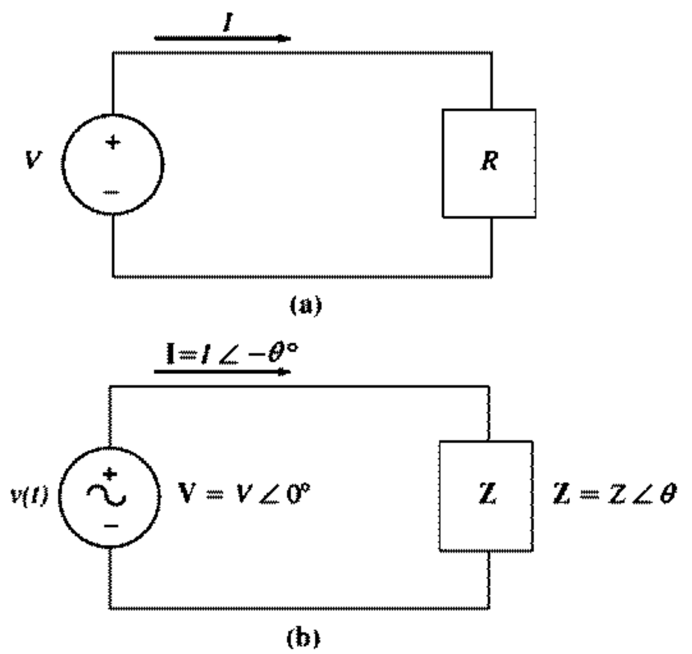


FIGURE 1–29
(a) A dc voltage source supplying a load with resistance R. (b) An ac voltage source supplying a load with impedance $\mathbf{Z} = Z \angle \theta\ \Omega$.

where V is the rms value of the voltage applied to the load, and the resulting current flow is

$$i(t) = \sqrt{2}I \cos(\omega t - \theta) \tag{1–57}$$

where I is the rms value of the current flowing through the load.

The instantaneous power supplied to this load at any time t is

$$p(t) = v(t)i(t) = 2VI \cos \omega t \cos(\omega t - \theta) \tag{1–58}$$

The angle θ in this equation is the *impedance angle* of the load. For inductive loads, the impedance angle is positive, and the current waveform lags the voltage waveform by θ degrees.

If we apply trigonometric identities to Equation (1–58), it can be manipulated into an expression of the form

$$p(t) = VI \cos \theta\,(1 + \cos 2\omega t) + VI \sin \theta \sin 2\omega t \tag{1–59}$$

The first term of this equation represents the power supplied to the load by the component of current that is *in phase* with the voltage, while the second term represents the power supplied to the load by the component of current that is *90° out of phase* with the voltage. The components of this equation are plotted in Figure 1–30.

Note that the *first* term of the instantaneous power expression is always positive, but it produces pulses of power instead of a constant value. The average value of this term is

$$P = VI \cos \theta \tag{1–60}$$

which is the *average* or *real* power (P) supplied to the load by term 1 of the Equation (1–59). The units of real power are watts (W), where 1 W = 1 V × 1 A.

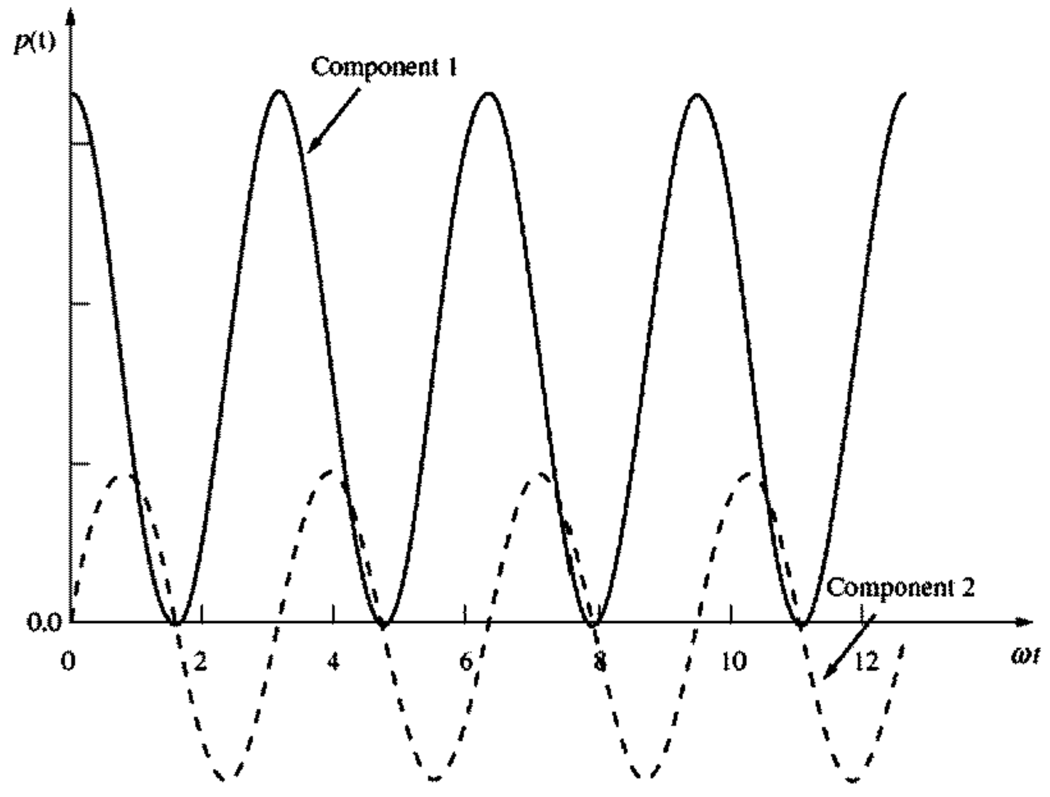


FIGURE 1–30
The components of power supplied to a single-phase load versus time. The first component represents the power supplied by the component of current *in phase* with the voltage, while the second term represents the power supplied by the component of current *90° out of phase* with the voltage.

Note that the *second* term of the instantaneous power expression is positive half of the time and negative half of the time, so that *the average power supplied by this term is zero.* This term represents power that is first transferred from the source to the load, and then returned from the load to the source. The power that continually bounces back and forth between the source and the load is known as *reactive power* (Q). Reactive power represents the energy that is first stored and then released in the magnetic field of an inductor, or in the electric field of a capacitor.

The reactive power of a load is given by

$$Q = VI \sin \theta \tag{1–61}$$

where θ is the impedance angle of the load. By convention, Q is positive for inductive loads and negative for capacitive loads, because the impedance angle θ is positive for inductive loads and negative for capacitive loads. The units of reactive power are volt-amperes reactive (var), where 1 var = 1 V × 1 A. Even though the dimensional units are the same as for watts, reactive power is traditionally given a unique name to distinguish it from power actually supplied to a load.

The apparent power (S) supplied to a load is defined as the product of the voltage across the load and the current through the load. This is the power that "appears" to be supplied to the load if the phase angle differences between voltage and current are ignored. Therefore, the apparent power of a load is given by

$$S = VI \tag{1–62}$$

The units of apparent power are volt-amperes (VA), where 1 VA = 1 V × 1 A. As with reactive power, apparent power is given a distinctive set of units to avoid confusing it with real and reactive power.

Alternative Forms of the Power Equations

If a load has a constant impedance, then Ohm's law can be used to derive alternative expressions for the real, reactive, and apparent powers supplied to the load. Since the magnitude of the voltage across the load is given by

$$V = IZ \tag{1–63}$$

substituting Equation (1–63) into Equations (1–60) to (1–62) produces equations for real, reactive, and apparent power expressed in terms of current and impedance:

$$P = I^2 Z \cos\theta \tag{1–64}$$

$$Q = I^2 Z \sin\theta \tag{1–65}$$

$$S = I^2 Z \tag{1–66}$$

where $|Z|$ is the magnitude of the load impedance Z.

Since the impedance of the load Z can be expressed as

$$Z = R + jX = |Z| \cos\theta + j|Z| \sin\theta$$

we see from this equation that $R = |Z| \cos\theta$ and $X = |Z| \sin\theta$, so the real and reactive powers of a load can also be expressed as

$$P = I^2 R \tag{1–67}$$

$$Q = I^2 X \tag{1–68}$$

where R is the resistance and X is the reactance of load Z.

Complex Power

For simplicity in computer calculations, real and reactive power are sometimes represented together as a *complex power* **S**, where

$$\mathbf{S} = P + jQ \tag{1–69}$$

The complex power **S** supplied to a load can be calculated from the equation

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* \tag{1–70}$$

where the asterisk represents the complex conjugate operator.

To understand this equation, let's suppose that the voltage applied to a load is $\mathbf{V} = V \angle \alpha$ and the current through the load is $\mathbf{I} = I \angle \beta$. Then the complex power supplied to the load is

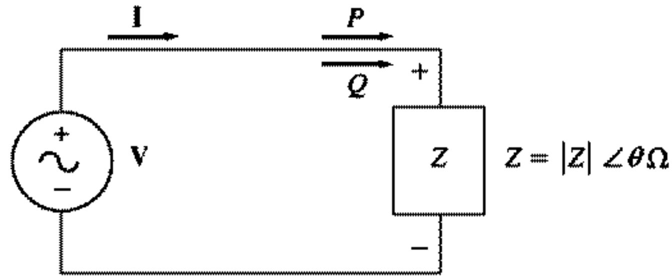


FIGURE 1–31
An inductive load has a *positive* impedance angle θ. This load produces a *lagging* current, and it consumes both real power P and reactive power Q from the source.

$$\begin{aligned}\mathbf{S} = \mathbf{VI}^* &= (V\angle\alpha)(I\angle -\beta) = VI\,\angle(\alpha - \beta) \\ &= VI\cos(\alpha - \beta) + jVI\sin(\alpha - \beta)\end{aligned}$$

The impedance angle θ is the difference between the angle of the voltage and the angle of the current ($\theta = \alpha - \beta$), so this equation reduces to

$$\begin{aligned}\mathbf{S} &= VI\cos\theta + jVI\sin\theta \\ &= P + jQ\end{aligned}$$

The Relationships between Impedance Angle, Current Angle, and Power

As we know from basic circuit theory, an inductive load (Figure 1–31) has a positive impedance angle θ, since the reactance of an inductor is positive. If the impedance angle θ of a load is positive, the phase angle of the current flowing through the load will *lag* the phase angle of the voltage across the load by θ.

$$\mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{V\angle 0^\circ}{|Z|\angle\theta} = \frac{V}{|Z|}\angle -\theta$$

Also, if the impedance angle θ of a load is positive, the reactive power consumed by the load will be positive (Equation 1–65), and the load is said to be consuming both real and reactive power from the source.

In contrast, a capacitive load (Figure 1–32) has a negative impedance angle θ, since the reactance of a capacitor is negative. If the impedance angle θ of a load is negative, the phase angle of the current flowing through the load will *lead* the phase angle of the voltage across the load by θ. Also, if the impedance angle θ of a load is negative, the reactive power Q consumed by the load will be *negative* (Equation 1–65). In this case, we say that the load is consuming real power from the source and *supplying* reactive power to the source.

The Power Triangle

The real, reactive, and apparent powers supplied to a load are related by the *power triangle*. A power triangle is shown in Figure 1–33. The angle in the lower left

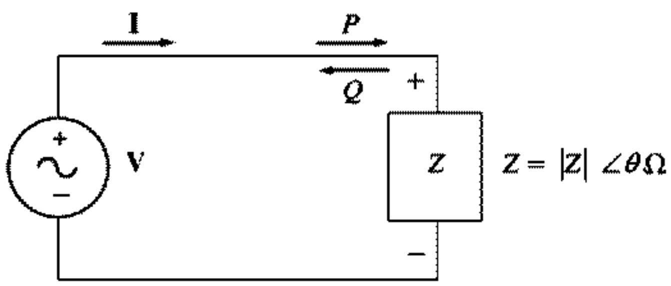
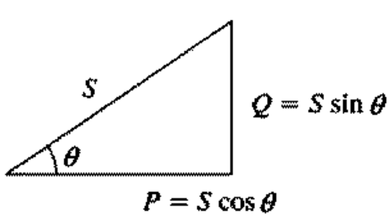


FIGURE 1–32
A capacitive load has a *negative* impedance angle θ. This load produces a *leading* current, and it consumes real power P from the source and while supplying reactive power Q to the source.



$$\cos\theta = \frac{P}{S}$$
$$\sin\theta = \frac{Q}{S}$$
$$\tan\theta = \frac{Q}{P}$$

FIGURE 1–33
The power triangle.

corner is the impedance angle θ. The adjacent side of this triangle is the real power P supplied to the load, the opposite side of the triangle is the reactive power Q supplied to the load, and the hypotenuse of the triangle is the apparent power S of the load.

The quantity $\cos\theta$ is usually known as the *power factor* of a load. The power factor is defined as the fraction of the apparent power S that is actually supplying real power to a load. Thus,

$$\text{PF} = \cos\theta \tag{1–71}$$

where θ is the impedance angle of the load.

Note that $\cos\theta = \cos(-\theta)$, so the power factor produced by an impedance angle of $+30°$ is exactly the same as the power factor produced by an impedance angle of $-30°$. Because we can't tell whether a load is inductive or capacitive from the power factor alone, it is customary to state whether the current is leading or lagging the voltage whenever a power factor is quoted.

The power triangle makes the relationships among real power, reactive power, apparent power, and the power factor clear, and provides a convenient way to calculate various power-related quantities if some of them are known.

Example 1–11. Figure 1–34 shows an ac voltage source supplying power to a load with impedance $Z = 20\angle -30°\ \Omega$. Calculate the current $\mathbf{I}$ supplied to the load, the power factor of the load, and the real, reactive, apparent, and complex power supplied to the load.

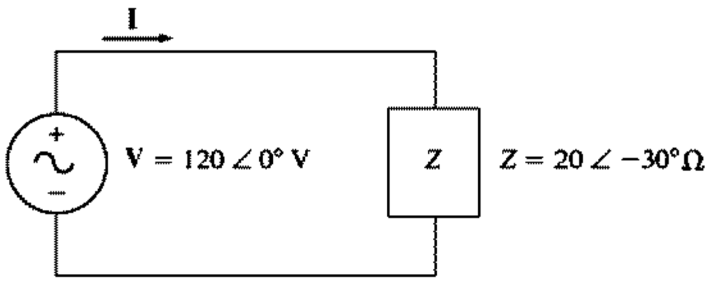


FIGURE 1–34
The circuit of Example 1–11.

Solution

The current supplied to this load is

$$\mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{120\angle 0°\ \text{V}}{20\angle -30°\ \Omega} = 6\angle 30°\ \text{A}$$

The power factor of the load is

$$\text{PF} = \cos\theta = \cos(-30°) = 0.866 \text{ leading} \tag{1–71}$$

(Note that this is a capacitive load, so the impedance angle θ is negative, and the current *leads* the voltage.)

The real power supplied to the load is

$$P = VI\cos\theta \tag{1–60}$$

$$P = (120\ \text{V})(6\ \text{A})\cos(-30°) = 623.5\ \text{W}$$

The reactive power supplied to the load is

$$Q = VI\sin\theta \tag{1–61}$$

$$Q = (120\ \text{V})(6\ \text{A})\sin(-30°) = -360\ \text{VAR}$$

The apparent power supplied to the load is

$$S = VI \tag{1–62}$$

$$Q = (120\ \text{V})(6\ \text{A}) = 720\ \text{VA}$$

The complex power supplied to the load is

$$\begin{aligned} \mathbf{S} &= \mathbf{VI}^* \qquad (1\text{–}70)\\ &= (120\angle 0°\ \text{V})(6\angle -30°\ \text{A})^* \\ &= (120\angle 0°\ \text{V})(6\angle 30°\ \text{A}) = 720\angle 30°\ \text{VA} \\ &= 623.5 - j360\ \text{VA} \end{aligned}$$

1.10 SUMMARY

This chapter has reviewed briefly the mechanics of systems rotating about a single axis and introduced the sources and effects of magnetic fields important in the understanding of transformers, motors, and generators.

Historically, the English system of units has been used to measure the mechanical quantities associated with machines in English-speaking countries.

Recently, the SI units have superseded the English system almost everywhere in the world except in the United States, but rapid progress is being made even there. Since SI is becoming almost universal, most (but not all) of the examples in this book use this system of units for mechanical measurements. Electrical quantities are always measured in SI units.

In the section on mechanics, the concepts of angular position, angular velocity, angular acceleration, torque, Newton's law, work, and power were explained for the special case of rotation about a single axis. Some fundamental relationships (such as the power and speed equations) were given in both SI and English units.

The production of a magnetic field by a current was explained, and the special properties of ferromagnetic materials were explored in detail. The shape of the magnetization curve and the concept of hysteresis were explained in terms of the domain theory of ferromagnetic materials, and eddy current losses were discussed.

Faraday's law states that a voltage will be generated in a coil of wire that is proportional to the time rate of change in the flux passing through it. Faraday's law is the basis of transformer action, which is explored in detail in Chapter 3.

A current-carrying wire present in a magnetic field, if it is oriented properly, will have a force induced on it. This behavior is the basis of motor action in all real machines.

A wire moving through a magnetic field with the proper orientation will have a voltage induced in it. This behavior is the basis of generator action in all real machines.

A simple linear dc machine consisting of a bar moving in a magnetic field illustrates many of the features of real motors and generators. When a load is attached to it, it slows down and operates as a motor, converting electric energy into mechanical energy. When a force pulls the bar faster than its no-load steady-state speed, it acts as a generator, converting mechanical energy into electric energy.

In ac circuits, the real power P is the average power supplied by a source to a load. The reactive power Q is the component of power that is exchanged back and forth between a source and a load. By convention, positive reactive power is consumed by inductive loads ($+\theta$) and negative reactive power is consumed (or positive reactive power is supplied) by capacitive loads ($-\theta$). The apparent power S is the power that "appears" to be supplied to the load if only the magnitudes of the voltages and currents are considered.

QUESTIONS

1–1. What is torque? What role does torque play in the rotational motion of machines?

1–2. What is Ampere's law?

1–3. What is magnetizing intensity? What is magnetic flux density? How are they related?

1–4. How does the magnetic circuit concept aid in the design of transformer and machine cores?

1–5. What is reluctance?

1–6. What is a ferromagnetic material? Why is the permeability of ferromagnetic materials so high?

1–7. How does the relative permeability of a ferromagnetic material vary with magnetomotive force?

1–8. What is hysteresis? Explain hysteresis in terms of magnetic domain theory.

1–9. What are eddy current losses? What can be done to minimize eddy current losses in a core?

1–10. Why are all cores exposed to ac flux variations laminated?

1–11. What is Faraday's law?

1–12. What conditions are necessary for a magnetic field to produce a force on a wire?

1–13. What conditions are necessary for a magnetic field to produce a voltage in a wire?

1–14. Why is the linear machine a good example of the behavior observed in real dc machines?

1–15. The linear machine in Figure 1–19 is running at steady state. What would happen to the bar if the voltage in the battery were increased? Explain in detail.

1–16. Just how does a decrease in flux produce an increase in speed in a linear machine?

1–17. Will current be leading or lagging voltage in an inductive load? Will the reactive power of the load be positive or negative?

1–18. What are real, reactive, and apparent power? What units are they measured in? How are they related?

1–19. What is power factor?

PROBLEMS

1–1. A motor's shaft is spinning at a speed of 3000 r/min. What is the shaft speed in radians per second?

1–2. A flywheel with a moment of inertia of 2 kg • m^2 is initially at rest. If a torque of 5 N • m (counterclockwise) is suddenly applied to the flywheel, what will be the speed of the flywheel after 5 s? Express that speed in both radians per second and revolutions per minute.

1–3. A force of 10 N is applied to a cylinder, as shown in Figure P1–1. What are the magnitude and direction of the torque produced on the cylinder? What is the angular acceleration α of the cylinder?

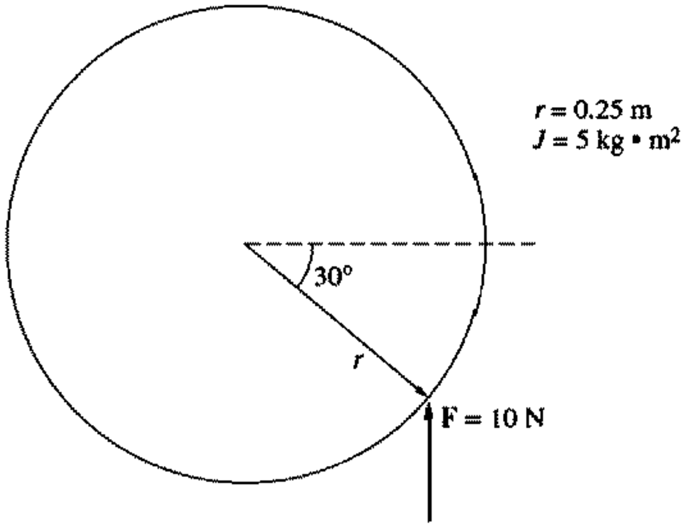


FIGURE P1–1
The cylinder of Problem 1–3.

1–4. A motor is supplying 60 N • m of torque to its load. If the motor's shaft is turning at 1800 r/min, what is the mechanical power supplied to the load in watts? In horsepower?

1–5. A ferromagnetic core is shown in Figure P1–2. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.005 Wb. With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 1000.

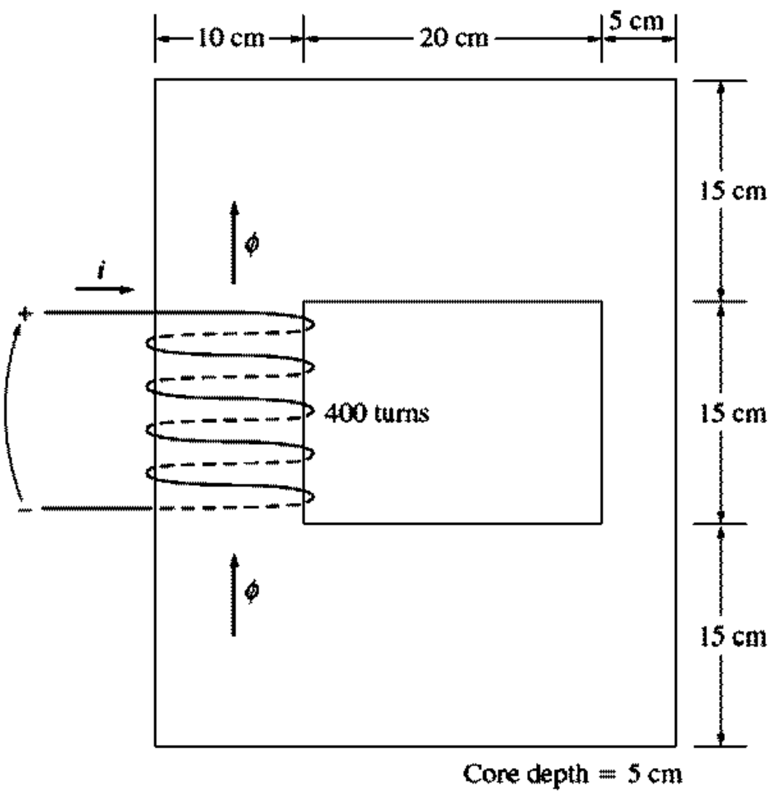


FIGURE P1–2
The core of Problems 1–5 and 1–16.

1–6. A ferromagnetic core with a relative permeability of 1500 is shown in Figure P1–3. The dimensions are as shown in the diagram, and the depth of the core is 7 cm. The air gaps on the left and right sides of the core are 0.070 and 0.050 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are 400 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?

1–7. A two-legged core is shown in Figure P1–4. The winding on the left leg of the core (N_1) has 400 turns, and the winding on the right (N_2) has 300 turns. The coils are wound in the directions shown in the figure. If the dimensions are as shown, then what flux would be produced by currents $i_1 = 0.5$ A and $i_2 = 0.75$ A? Assume $\mu_r = 1000$ and constant.

1–8. A core with three legs is shown in Figure P1–5. Its depth is 5 cm, and there are 200 turns on the leftmost leg. The relative permeability of the core can be assumed to be 1500 and constant. What flux exists in each of the three legs of the core? What is the flux density in each of the legs? Assume a 4 percent increase in the effective area of the air gap due to fringing effects.

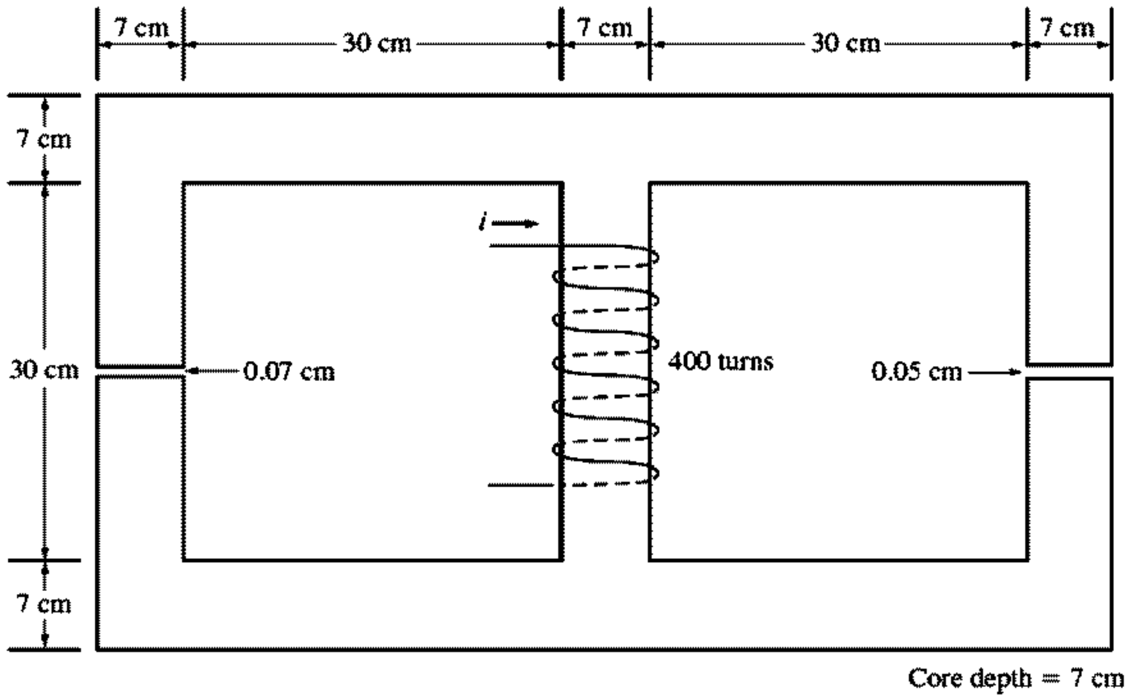


FIGURE P1–3
The core of Problem 1–6.

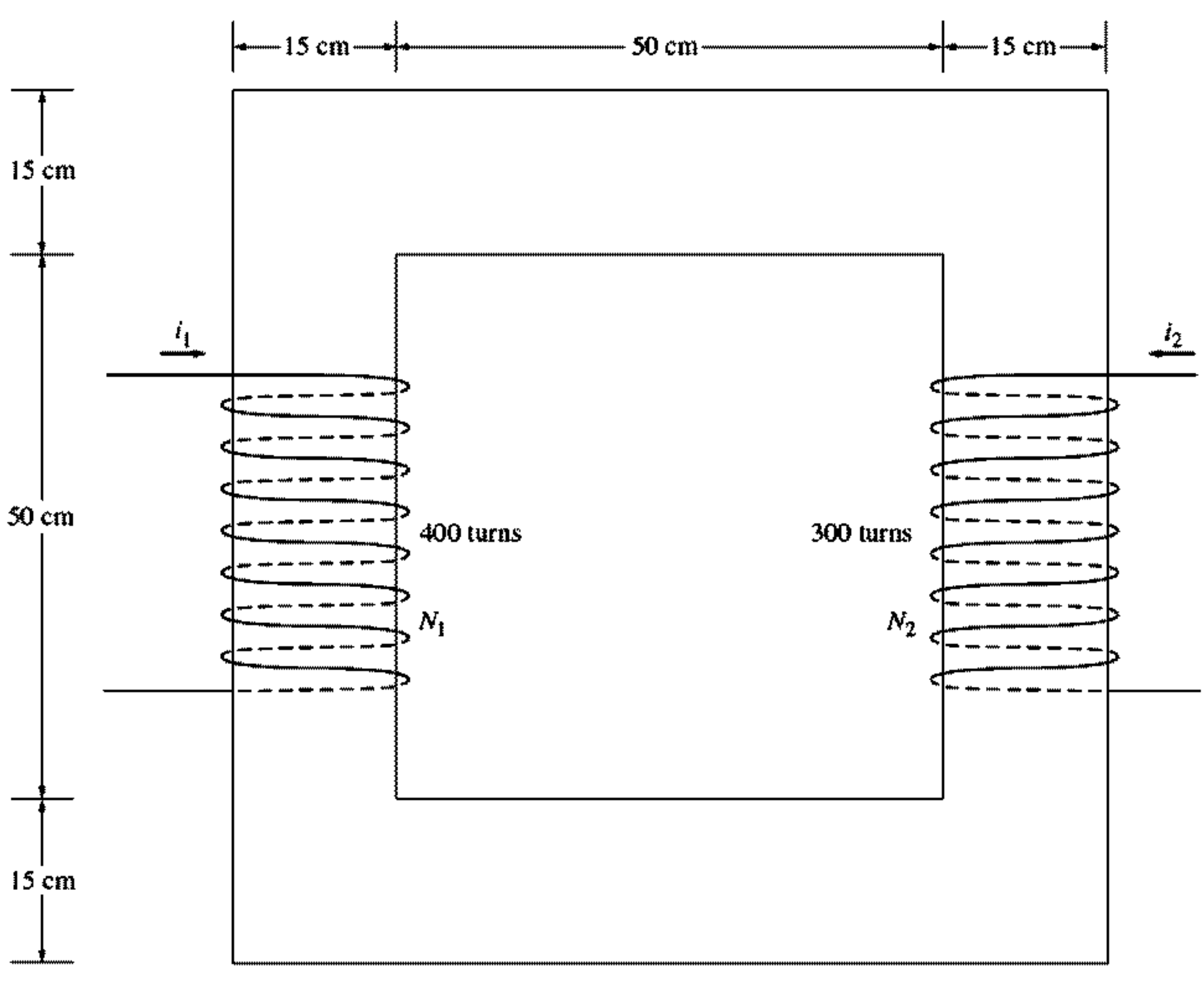


FIGURE P1–4
The core of Problems 1–7 and 1–12.

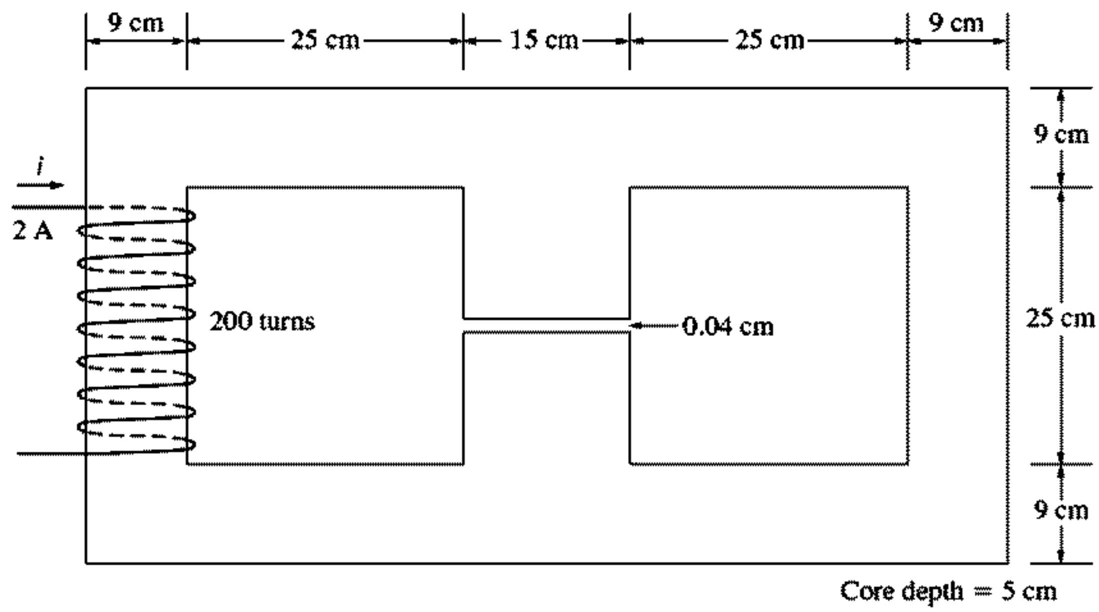


FIGURE P1–5
The core of Problem 1–8.

1–9. The wire shown in Figure P1–6 is carrying 5.0 A in the presence of a magnetic field. Calculate the magnitude and direction of the force induced on the wire.

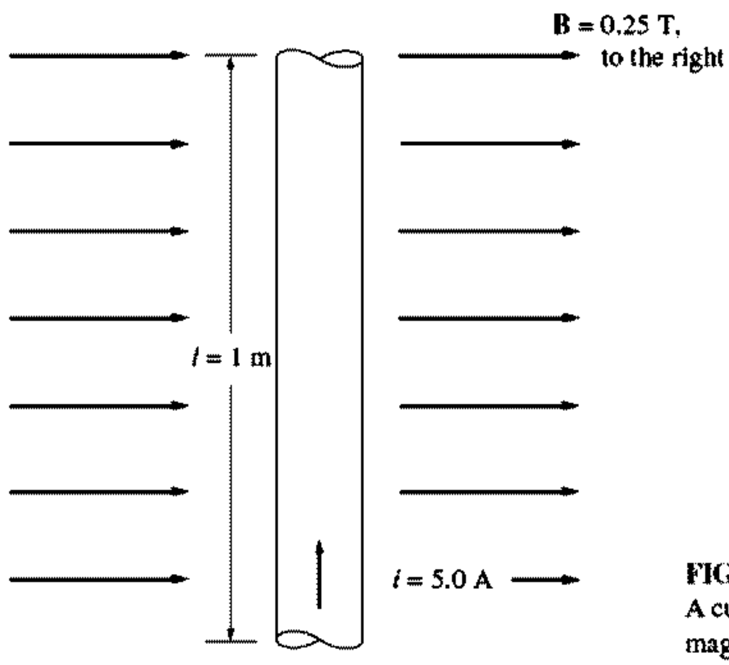


FIGURE P1–6
A current-carrying wire in a magnetic field (Problem 1–9).

1–10. The wire shown in Figure P1–7 is moving in the presence of a magnetic field. With the information given in the figure, determine the magnitude and direction of the induced voltage in the wire.

1–11. Repeat Problem 1–10 for the wire in Figure P1–8.

1–12. The core shown in Figure P1–4 is made of a steel whose magnetization curve is shown in Figure P1–9. Repeat Problem 1–7, but this time do *not* assume a constant value of μ_r. How much flux is produced in the core by the currents specified? What is the relative permeability of this core under these conditions? Was the assumption

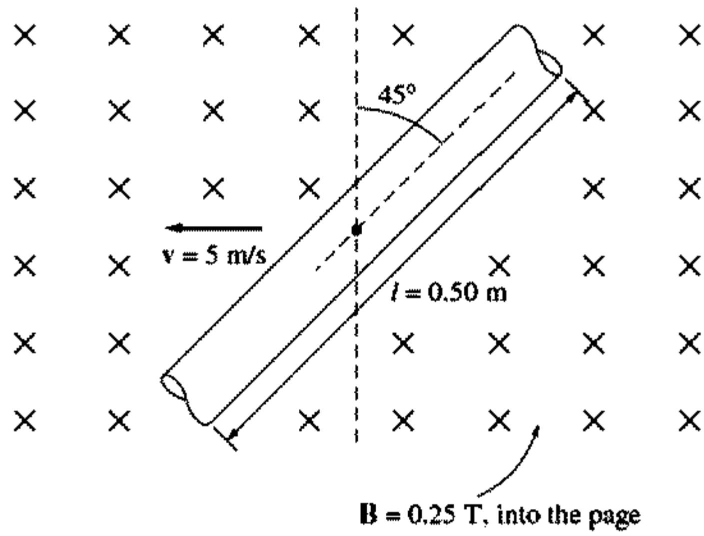


FIGURE P1–7
A wire moving in a magnetic field (Problem 1–10).

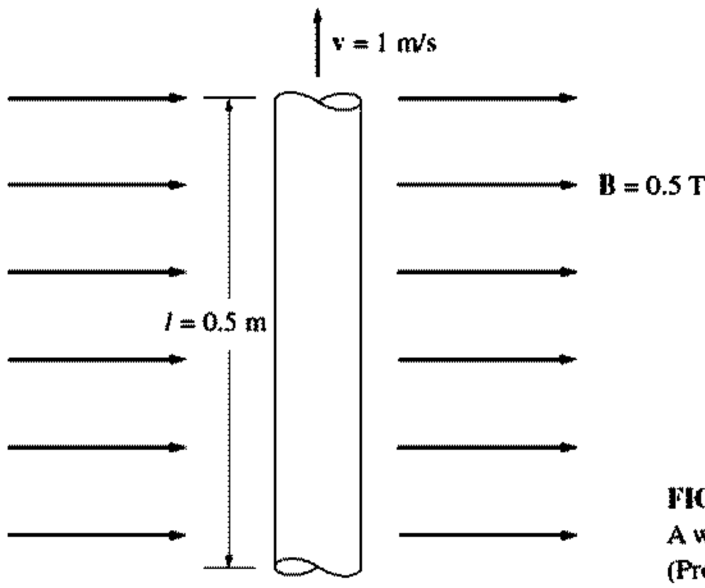


FIGURE P1–8
A wire moving in a magnetic field (Problem 1–11).

in Problem 1–7 that the relative permeability was equal to 1000 a good assumption for these conditions? Is it a good assumption in general?

1–13. A core with three legs is shown in Figure P1–10. Its depth is 8 cm, and there are 400 turns on the center leg. The remaining dimensions are shown in the figure. The core is composed of a steel having the magnetization curve shown in Figure 1–10c. Answer the following questions about this core:

(*a*) What current is required to produce a flux density of 0.5 T in the central leg of the core?

(*b*) What current is required to produce a flux density of 1.0 T in the central leg of the core? Is it twice the current in part (*a*)?

(*c*) What are the reluctances of the central and right legs of the core under the conditions in part (*a*)?

(*d*) What are the reluctances of the central and right legs of the core under the conditions in part (*b*)?

(*e*) What conclusion can you make about reluctances in real magnetic cores?

1–14. A two-legged magnetic core with an air gap is shown in Figure P1–11. The depth of the core is 5 cm, the length of the air gap in the core is 0.06 cm, and the number of turns on the coil is 1000. The magnetization curve of the core material is shown in

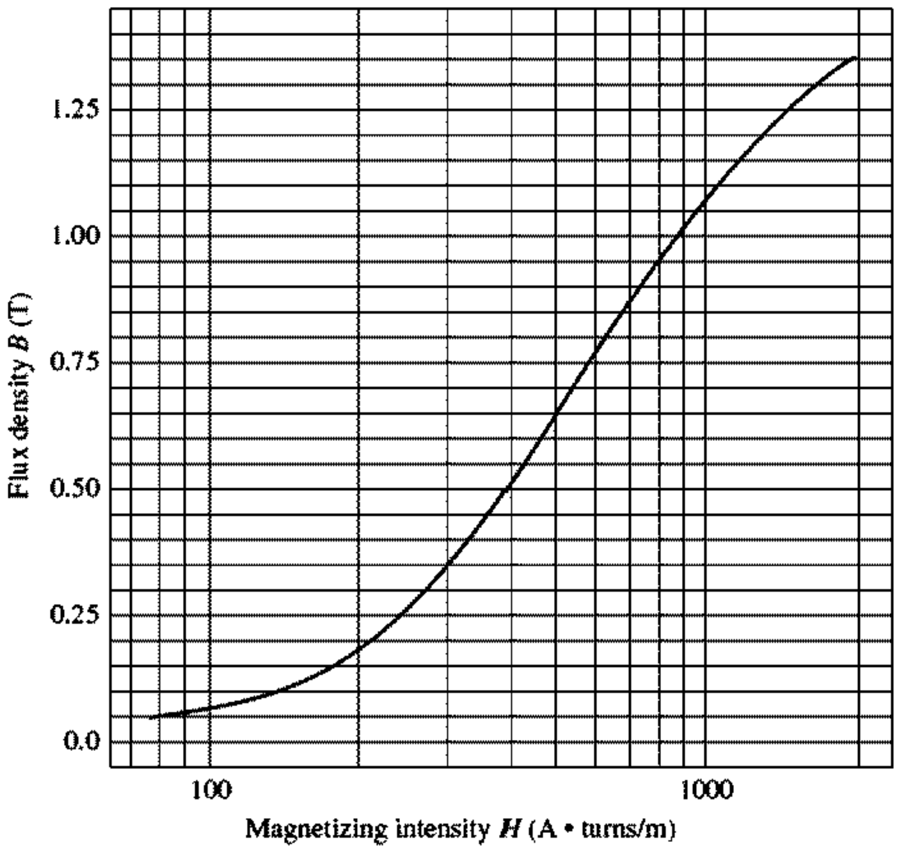


FIGURE P1–9
The magnetization curve for the core material of Problems 1–12 and 1–14.

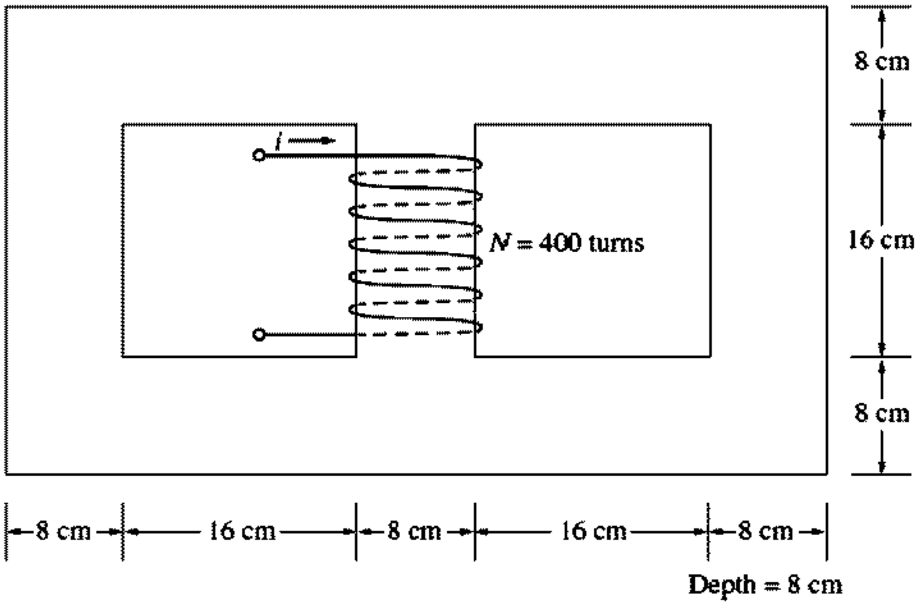


FIGURE P1–10
The core of Problem 1–13.

Figure P1–9. Assume a 5 percent increase in effective air-gap area to account for fringing. How much current is required to produce an air-gap flux density of 0.5 T? What are the flux densities of the four sides of the core at that current? What is the total flux present in the air gap?

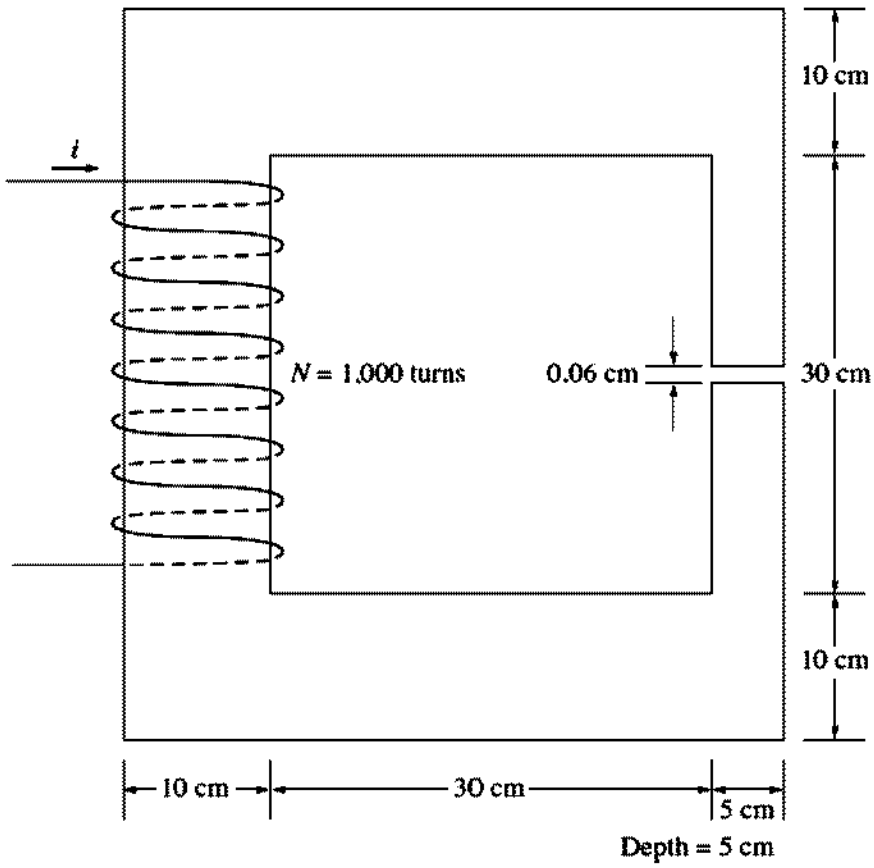


FIGURE P1–11
The core of Problem 1–14.

1–15. A transformer core with an effective mean path length of 10 in has a 300-turn coil wrapped around one leg. Its cross-sectional area is 0.25 in^2, and its magnetization curve is shown in Figure 1–10c. If current of 0.25 A is flowing in the coil, what is the total flux in the core? What is the flux density?

1–16. The core shown in Figure P1–2 has the flux ϕ shown in Figure P1–12. Sketch the voltage present at the terminals of the coil.

1–17. Figure P1–13 shows the core of a simple dc motor. The magnetization curve for the metal in this core is given by Figure 1–10c and d. Assume that the cross-sectional area of each air gap is 18 cm^2 and that the width of each air gap is 0.05 cm. The effective diameter of the rotor core is 4 cm.

(a) It is desired to build a machine with as great a flux density as possible while avoiding excessive saturation in the core. What would be a reasonable maximum flux density for this core?

(b) What would be the total flux in the core at the flux density of part *(a)*?

(c) The maximum possible field current for this machine is 1 A. Select a reasonable number of turns of wire to provide the desired flux density while not exceeding the maximum available current.

1–18. Assume that the voltage applied to a load is $\mathbf{V} = 208\angle -30°$ V and the current flowing through the load is $\mathbf{I} = 5\angle 15°$ A.

(a) Calculate the complex power **S** consumed by this load.

(b) Is this load inductive or capacitive?

(c) Calculate the power factor of this load.

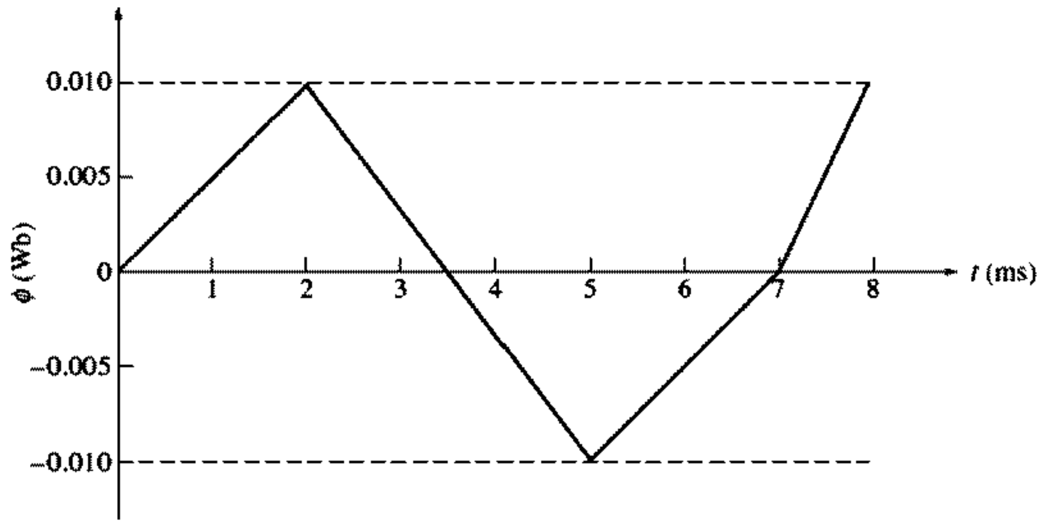


FIGURE P1–12
Plot of flux ϕ as a function of time for Problem 1–16.

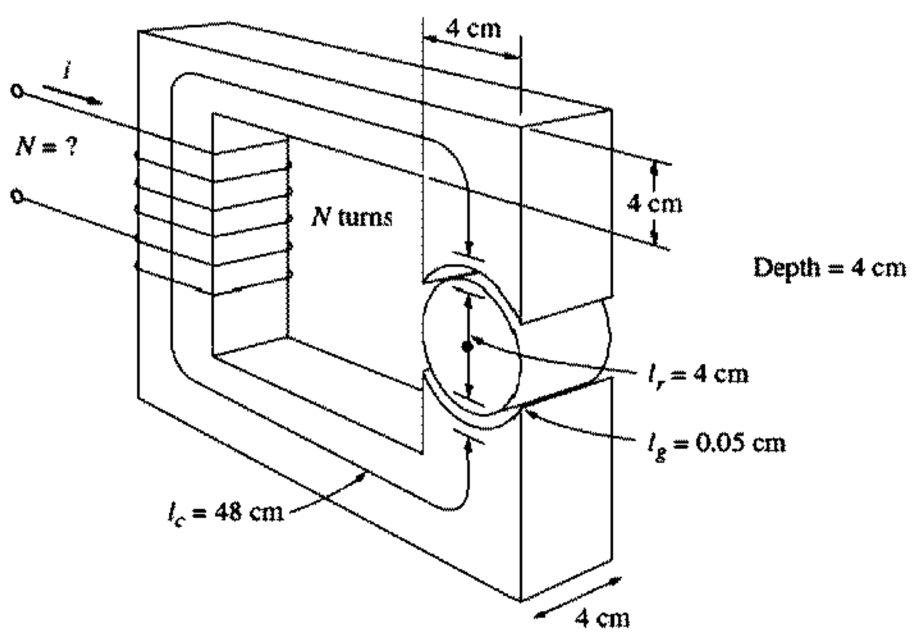


FIGURE P1–13
The core of Problem 1–17.

(*d*) Calculate the reactive power consumed or supplied by this load. Does the load consume reactive power from the source or supply it to the source?

1–19. Figure P1–14 shows a simple single-phase ac power system with three loads. The voltage source is $\mathbf{V} = 120\angle 0^\circ$ V, and the impedances of the three loads are

$$Z_1 = 5\angle 30^\circ\ \Omega \qquad Z_2 = 5\angle 45^\circ\ \Omega \qquad Z_3 = 5\angle -90^\circ\ \Omega$$

Answer the following questions about this power system.

(*a*) Assume that the switch shown in the figure is open, and calculate the current $\mathbf{I}$, the power factor, and the real, reactive, and apparent power being supplied by the load.

(*b*) Assume that the switch shown in the figure is closed, and calculate the current **I**, the power factor, and the real, reactive, and apparent power being supplied by the load.

(*c*) What happened to the current flowing from the source when the switch closed? Why?

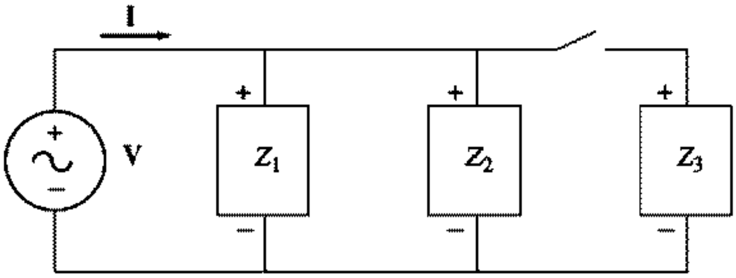


FIGURE P1–14
The circuit of Problem 1–19.

1–20. Demonstrate that Equation (1–59) can be derived from Equation (1–58) using the simple trigonometric identities:

$$p(t) = v(t)i(t) = 2VI\cos\omega t\cos(\omega t - \theta) \qquad (1\text{–}58)$$

$$p(t) = VI\cos\theta\,(1 + \cos 2\omega t) + VI\sin\theta\sin 2\omega t \qquad (1\text{–}59)$$

1–21. The linear machine shown in Figure P1–15 has a magnetic flux density of 0.5 T directed into the page, a resistance of 0.25 Ω, a bar length l = 1.0 m, and a battery voltage of 100 V.

(*a*) What is the initial force on the bar at starting? What is the initial current flow?

(*b*) What is the no-load steady-state speed of the bar?

(*c*) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady-state speed? What is the efficiency of the machine under these circumstances?

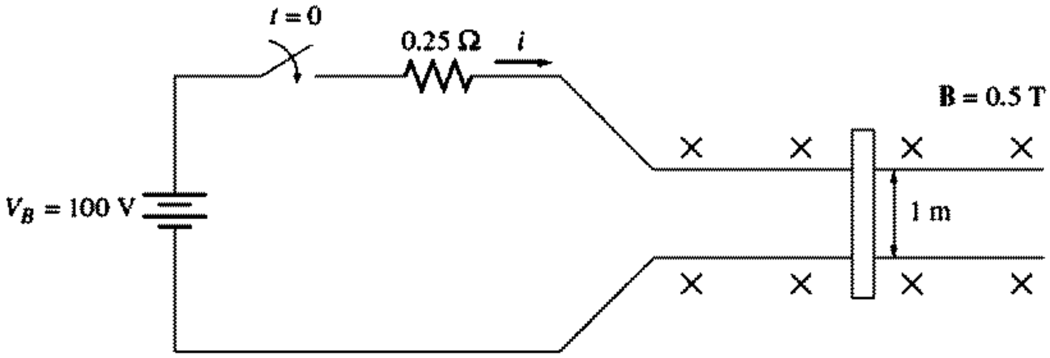


FIGURE P1–15
The linear machine in Problem 1–21.

1–22. A linear machine has the following characteristics:

$$\mathbf{B} = 0.33\text{ T into page} \qquad R = 0.50\ \Omega$$

$$l = 0.5\text{ m} \qquad V_B = 120\text{ V}$$

(*a*) If this bar has a load of 10 N attached to it opposite to the direction of motion, what is the steady-state speed of the bar?

(*b*) If the bar runs off into a region where the flux density falls to 0.30 T, what happens to the bar? What is its final steady-state speed?

(*c*) Suppose V_B is now decreased to 80 V with everything else remaining as in part *b*. What is the new steady-state speed of the bar?

(*d*) From the results for parts *b* and *c*, what are two methods of controlling the speed of a linear machine (or a real dc motor)?

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